Chapter 6

AP Stat

Simulations: Imitation of chance behavior based on a model that accurately reflects a situation

Cards, dice, random number generator/table, etc

When Performing a Simulation:

1. State the question/Problem of interest.

2. Explain how you plan on executing a simulation (in detail) that will match the chance behavior being investigated. Include how many repetitions, what corresponds to what, and so on

3. Perform the Simulation. Display results (graphs, etc).

4. Use the results to answer the question of interest.

AP note: On the AP, students may receive full credit on a question using a simulation even if the problem does not call for it. This is why it is a good thing to know/study

***AP exam common error:** When working with a random digit table, be sure to clearly communicate your method that you are using. You need to use the same length digits: EX: labeling 1500 items, use 0001,0002,......1500.(use 4 digits consistently.*

Example:

At a (college) department picnic, 18 students in mathematics/statistics department decide to play a softball game. Twelve of the 18 students are math majors. 6 are statistics majors. To divide into two teams of 9, one of the professors put all the players names in a hat and drew out 9 players to form a team. The players were surprised that one team was made up of entirely math majors. Is it possible that the names weren't adequately mixed in the hat, or could this happen by chance? Design and carry out a simulation to help answer this question.

State Problem/Question: what is the probability for a team of 9 to be entirely M.M. w/ 12 MM and 6 SM. Being drawn from a hat

Plan:
(include how many trials) Put 18 pieces (equal) of paper w/ 12 M and 6 S folded into a cup dump the cup & split into 2 piles of 9

Do:

Results/Conclusion with sim showing 0% chance of getting all M.M., That it calls the hat mix questionable

Example:

In an attempt to increase sales, a breakfast cereal company decides to offer a NASCAR promotion. Each box of cereal will contain a collectable card featuring one of these NASCAR drivers: Jeff Gordon, Dale Earnhardt Jr., Tony Stewart, Danica Patrick, or Jimmie Johnson. The company says that each of the five cards is equally likely to appear in any box of cereal. A NASCAR fan decides to keep buying boxes of cereal until she has all 5 drivers' cards. She is surprised when it takes her 23 boxes to get the full set of cards. Should she be surprised? Design and carry out a simulation to help answer the question. (use rand # generator)

State question: whats the probability that it will take 23 or more boxes of cereal to get all 5 cards?

Plan (include details) rand Int #'s 1-5. one trial is complete when # 1-5 appear
randint(1, 5) 3 2 1 4 3 5 = 6

Do: (include graphical rep)

Conclusion: After 189 trials, 10% showed having to ~~by~~ buy more than 23 boxes This is a low chance, so yes, she should be suprised

DEC 22

AP STAT

1. Notes Quiz- socrative HKREILLY
2. Get papers back.
3. Pick up worksheets for over break

Jan 2, 2018- WELCOME BACK

Objective: (*1)Students will develop simulations to make predictions. (2) Students will explore the idea of randomness and the myth associated with it*

1. HW discussion/review....a Prelude to Expected Value.
2. Randomness Activity
3. Check calendar for HW

QUIZ FRIDAY on SIMULATIONS

Remember your Project proposals are due friday 1/5. They can be emailed to me

Jan 3

AP Stat

QUIZ FRIDAY-Simulations

Project Proposal Due Friday- EMAIL

Objective: Students will determine expected values sample space and other probability vocabulary.

1. Warm up: Love Is Blind Revisited
2. Rev HW answers- see handout
3. Review of notes 6.2,and then some ......see how far we get
4. Check Calendar for HW (pg 423 etc)

Note: plan ahead for snow day......bring home your study materials

Expected value is just how it sounds. It is what to "expect" in the long run

When we talk about probability distributions (remember that means table) the **EXPECTED VALUE** is essentially the **MEAN** of the probability distribution. These will get used interchangeably.

For example:

Roulette

On an American Roulette wheel, there are 38 slots numbered 1-36, 0, and 00. Half the slots 1-36 are red and the other half are black. The 0 and 00 are green. Suppose a player places a simple $1 bet on RED. If it lands Red then they win a dollar. If it lands in any other then they lose there $1.

Let's define the random variable X=net gain from a single $1 bet on red.

The possible values of X are -$1 or +1.
The chance that the ball lands on red is 18/38

The Chance that the ball lands on another color is 20/38

Value	-$1	+	$1
Probability	20/38		18/38

(fill this in)

What is the players average gain? (long run average)

$$-1\left(\frac{20}{38}\right) + 1\left(\frac{18}{38}\right) = -.0\overline{5}$$

(Think about it it.......if they played 38 times........how many expected wins? Losses? Do they have a net gain or loss? This average is the expected loss per bet over the long run)

Jan 3

Probability

is the Quantification of chance behavior- over the long run.
(sometimes referred to as long term relative frequency)

- Probability is between 0 and 1. It can not be greater than 1. Describes the proportion of times the outcome would occur in a long series of repetitions.

Random in statistics refers to uncertain outcome- unpredictable behavior.

An Event in statistics is an outcome or set of outcomes of random phenomina

- For example: Toss a coin 25 times and determine the probability of getting an H- the EVENT is the # heads from total tosses. The sample space is the total # of tosses. Or from a deck of 52 cards, what is the probabiluty of drawing a queen. Drawing the queen is the Event- the sample space is the total #cards

Myths about randomness:

Our (human) intuition about randomness tries to tell us it should be predictable- like it should look a certain way- as seen in the activity.

Myth about "law of averages"

Believers in the law of averages, expect that after muliple events with the same result, the run/streak should break prducing a different outcome. For example if a coin was tossed 6 times and came up TTTTTT then the believer in the law ov averages thinks and H should be the next result. In fact, not the case. the myth is that future outcomes must make up for the imbalance. (in the LONG run of course things balance out but does not impact short run events)

(Actuarial work-life insurance- studies the random life behaviors/ phenomena- predictability)

Theoretical/Classical Probability- in theory what should the probability be EX coin toss

Empirical/Experimental Probability: The probability generated from performing an experiment over and over.

Do coin toss in simulation app here

Let's back up to SAMPLE SPACE.

Sample space is the set of all possible outcomes.

To determine sample space, there are three methods:

- Tree diagram (if not a broad range of possibilities
- Multiplication rule (easy and good for large sample spaces)(this is the counting principle
- Making a list

EXAMPLE: rolling a die and flipping a coin, what is the sample space

Tree Diagram:

1 < H T
2 < H T
3 < H T
4 < H T
5 < H T
6 < H T

12

Mult Rule:

$6 \cdot 2 = 12$

List:

1 H
1 T
2 H
2 T

Another Example:

Flip 2 coins and roll a die. What is the Sample Space?

$2 \cdot 2 \cdot 6 = 24$

Another example:

There are 5 appetizers, 6 different salads and 14 entrees on a menu.

A)What is the total number of possible outcomes if one of each is selected.

$5 \cdot 6 \cdot 14 = 420$

B) 2 different appetizers, one salad and 2 different entrees are selected. What is the total number of possibilities.

$5 \cdot 4 \cdot 6 \cdot 14 \cdot 13 = 21{,}840$

C) 2 appetizers, one salad and 2 entrees (that can repeat). What is the total number of possibilities now?

$5 \cdot 5 \cdot 6 \cdot 14 \cdot 14 = 29{,}400$

**Sampling with replacement vs without replacement

Other Important Info about Probability: (rules)

- Sum of probabilities must add to 1 (probability distribution)
- Mutually Exclusive Events aka **DISJOINT**: have NO possible outcomes the same. For example from a deck of cards, drawing a king or drawing a queen. A king can not be a queen.
- Mutually Inclusive Events: have "overlap" or can occur at the same time. For example from a deck of cards, drawing a king or a heart. Since a king can be a heart, this must be considered (subtracted out) when finding probability.

Other helpful things/Formulas:

AND: if you see AND (multiple events)- multiply probabilities Multiplication Rule for Independent Events: P(A and B)= P(A)P(B)

OR: if you see OR- add your probabilities: P(A or B)= P(A) + P(B) for Mutually exclusive (disjoint)

$$P(A \text{ or } B) = P(A) + P(B) - P(A\cap B) \text{ (inclusive/joint)}$$

The complement or 1- P(A) is the probability the event does not occur.

Notation: $P(A^c)$. Sometimes the complement is indicated with a tick mark superscript.

Ex: The probability of being color blind is 1/1000. The probability of not being color blind is 999/1000

(Basic)Exs: Standard deck of cards.

DISJ

A) What is the probability of drawing an Ace or a king

$$P(A \text{ or } K) = P(A) + P(K)$$
$$\frac{4}{52} + \frac{4}{52} = \frac{8}{52}$$

B) What is the probability of drawing an Ace or a spade

$$P(A \text{ or } S) = P(A) + P(S) - P(AS)$$
$$\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

C) What is the probability of drawing a jack, then queen, then 10 (without replacement)

$$P(J)\,P(Q)\,P(10)$$
$$\frac{4}{52} \cdot \frac{4}{51} \cdot \frac{4}{50} = .00048$$

D) What is the probability of drawing a Jack, and then queen and then 10 (with replacement)

$$P(J)\,P(Q) \cdot P(10)$$
$$\frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = .00046$$

Events that occur in a sequence

Conditional Probability: probability of an event occurringgiven another event has already occurred. The probability of even B occurring after event A has occurred: notation: P(B|A) "probability of B given A."

(Basic)Ex 1.
Two cards are selected from a standard deck of cards. What is the probability that the second card will be a queen given that the first card is a King? $\frac{4}{51}$

(Basic)Ex 2.
Two cards selected again (in succession). What is the probability that the second card will be an Ace, given the first card is an Ace? $\frac{3}{51}$

Multiple independent events. *Multiply each event.*
Example:
The probability that a salmon swims successfully through a dam is 0.85.

A)Find the probability that two salmons swim successfully through that dam.

$P(S)\,P(S)$

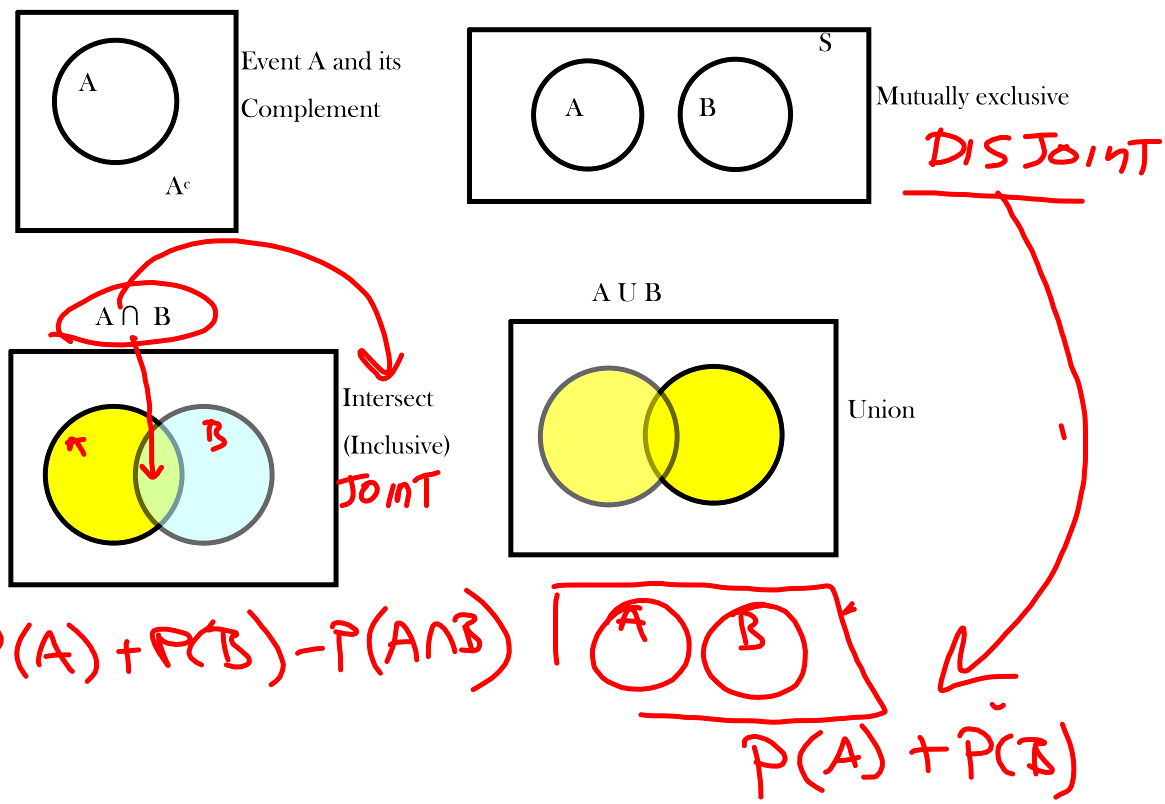
$(.85)(.85) = .7225$

B)Then find the probability that NEITHER swim successfully through the dam.

$P(NS)\,P(NS)$

$(.15)(.15) = .0225$

VENN DIAGRAMS



Let's get some notation squared away...because it can be confusing

Two events, A and B, are disjoint if they are mutually exclusive; i.e., if A∩B=Φ (nothing in common of overlapping).

If A and B are disjoint, then P(AUB)=P(A)+P(B) (this is OR) (if inclusive/joint you have to subtract the overlap which is P(A and B) or P(A)P(B))

Here is an example: $P(A \cap B)$

Event A has .6 chance of occuring

Event B has .3 chance of occuring. .6 .3

If disjoint, then P(AUB)= P(A)+P(B)

or it can be asked with "what is the probability that A or B occurs.

.6 +.3=.9 $P(A \cap B) = P(A)P(B)$

Now suppose they are joint (inclusive)

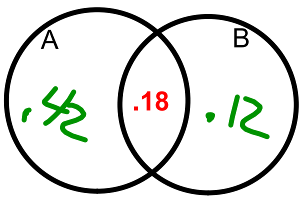
P(AUB)= P(A)+P(B) - P(A∩B)

= .6 + .3 - (.6x.3)

= .6 +.3 - .18

= .72

now what does this mean in making a venn diagram?



so, probability of A or B occurring means P(AUB)

Then think, intersection $\cap$ as "and"

like P(A and B) (both holding true) would be $P(A \cap B)$- the overlap section of the Venn diagram

In **conditional probability**, (be it one or multiple events occurring)

$P(A \cap B) = P(A)P(B|A)$- *another version of this is given on the next slide.*

these formulas are mostly about getting the notation to use them effectively.

EX.

What the probability when drawing 2 cards w/o replacement you get an Ace and then a king.

More conditional Probability "stuff"........oh what fun!

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

$P(A \cap B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Probability of B given A is the Prob of both divided by Prob of A

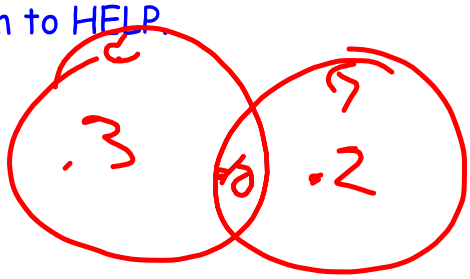
EX. Music styles other than rock and pop are becoming more popular. A survey of collage students finds that 40% like country music, 30% like gospel and 10% like both.

A)What is the conditional probability that a student likes gospel if we know they like country music?

$$P(G|C) = \frac{P(G \text{ and } C)}{P(C)} = \frac{.1}{.4} = .25$$

B) What is the conditional probability that a student who does not like country likes gospel. Use a Venn Diagram to HELP.

.2



EX:

According to the National Center for Health Statistics, in Dec 2008, 78% of US households has a traditional landline telephone. 80% of households had cell phones and 60% had both. Suppose we randomly select a household in December 2008.

A) Make a two way table that displays the sample space of this chance process.

B) Construct a Venn Diagram

C) Find the probability that the household has at least one of the two types of phones

D) Find the probability that the household has a cell phone only.

Mutual Independence

Roulette wheel Example

Consider a roulette wheel that has 36 numbers colored red (R) or black (B) according to the following pattern:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
R	R	R	R	R	B	B	B	B	R	R	R	R	B	B	B	B	B
36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19

and define the following three events:

Let A be the event that a spin of the wheel yields a RED number = {1, 2, 3, 4, 5, 10, 11, 12, 13, 24, 25, 26, 27, 32, 33, 34, 35, 36}.

Let B be the event that a spin of the wheel yields an EVEN number = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36}.

Let C be the event that a spin of the wheel yields a number no greater than 18 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18}.

Now consider the following two questions:

Are the events A, B, and C "pairwise independent?" That is, is event A independent of event B; event A independent of event C; and B independent of event C?

Does $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$?

https://www.youtube.com/watch?v=ZSnR_eX_3kg&feature=youtu.be

DETERMINING INDEPENDENCE

There are more formal ways to quantify dependent or independent events. You'll come across these formulas in basic probability.

P(A|B) = P(A).

P(B|A) = P(B)

The probability of A, given that B has happened, is the same as the probability of A. Likewise, the probability of B, given that A has happened, is the same as the probability of B. This shouldn't be a surprise, as one event doesn't affect the other.

Here's another way to look at it

$\boxed{\text{if } P(A \cap B) = P(A)\,P(B).}$..then the events are independent.****

EX. Suppose that for a group of consumers, the probability of eating pretzels is 0.75 and that the probability of drinking Coke is 0.65. Further suppose that the probability of eating pretzels and drinking Coke is 0.55. Determine if these two events are independent.

P(P and C) P(P) P(C)

P(P ∩ C) (.75)(.65)

.55 ≠ .49

Therefore not independ. Events

P(L & C) P(L) P(C)

.6 .6~~2~~4