12.3 An Introduction to Probability

What you should learn

GOAL 1 Find theoretical and experimental probabilities.

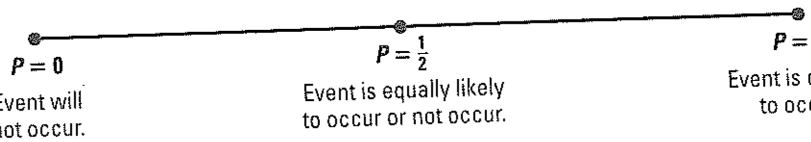
GOAL 2 Find geometric probabilities, as applied in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the probability that an archer hits the center of a target in **Ex. 46**.

GOAL 1 THEORETICAL AND EXPERIMENTAL PROBABILITY

The **probability** of an event is a number between 0 and 1 that indicates the likelihood the event will occur. An event that is certain to occur has a probability of 1. An event that *cannot* occur has a probability of 0. An event that is equally likely to occur or not occur has a probability of $\frac{1}{2}$.



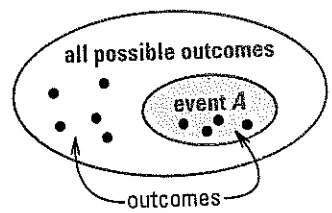
There are two types of probability: *theoretical* and *experimental*. Theoretical probability is defined below and experimental probability is defined on page 717.

THE THEORETICAL PROBABILITY OF AN EVENT

When all outcomes are equally likely, the **theoretical probability** that an event A will occur is:

$$P(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$$

The theoretical probability of an event is often simply called the probability of the event.



$P(A) = \frac{4}{9}$

EXAMPLE 1 *Finding Probabilities of Events*

You roll a six-sided die whose sides are numbered from 1 through 6. Find the probability of (**a**) rolling a 4, (**b**) rolling an odd number, and (**c**) rolling a number less than 7.

SOLUTION

a. Only one outcome corresponds to rolling a 4.

$$P(\text{rolling a 4}) = \frac{\text{number of ways to roll a 4}}{\text{number of ways to roll the die}} = \frac{1}{6}$$

b. Three outcomes correspond to rolling an odd number: rolling a 1, 3, or 5.

$$P(\text{rolling an odd number}) = \frac{\text{number of ways to roll an odd number}}{\text{number of ways to roll the die}} = \frac{3}{6} = \frac{1}{2}$$

c. All six outcomes correspond to rolling a number less than 7.

$$P(\text{rolling less than 7}) = \frac{\text{number of ways to roll less than 7}}{\text{number of ways to roll the die}} = \frac{6}{6} = 1$$

You can express a probability as a fraction, a decimal, or a percent. For instance, in part (b) of Example 1 the probability of rolling an odd number can be written as $\frac{1}{2}$, 0.5, or 50%.

REAL LIFE Music

EXAMPLE 2 *Probabilities Involving Permutations or Combinations*

You put a CD that has 8 songs in your CD player. You set the player to play the songs at random. The player plays all 8 songs without repeating any song.

a. What is the probability that the songs are played in the same order they are listed on the CD?

b. You have 4 favorite songs on the CD. What is the probability that 2 of your favorite songs are played first, in any order?

SOLUTION

a. There are 8! different *permutations* of the 8 songs. Of these, only 1 is the order in which the songs are listed on the CD. So, the probability is:

$$P(\text{playing 8 in order}) = \frac{1}{8!} = \frac{1}{40{,}320} \approx 0.0000248$$

STUDENT HELP

Skills Review
For help with converting decimals, fractions, and percents, see p. 906.

b. There are ${}_8C_2$ different *combinations* of 2 songs. Of these, ${}_4C_2$ contain 2 of your favorite songs. So, the probability is:

$$P(\text{playing 2 favorites first}) = \frac{{}_4C_2}{{}_8C_2} = \frac{6}{28} = \frac{3}{14} \approx 0.214$$

. . . .

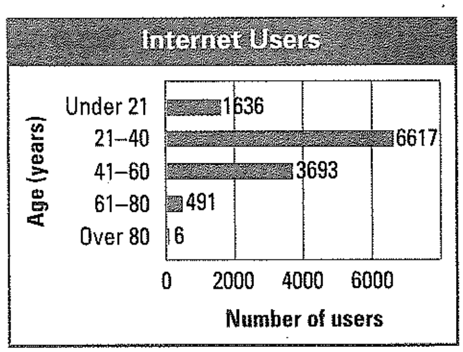
Sometimes it is not possible or convenient to find the theoretical probability of an event. In such cases you may be able to calculate an **experimental probability** by performing an experiment, conducting a survey, or looking at the history of the event.

REAL LIFE Internet

EXAMPLE 3 *Finding Experimental Probabilities*

In 1998 a survey asked Internet users for their ages. The results are shown in the bar graph. Find the experimental probability that a randomly selected Internet user is (**a**) at most 20 years old, and (**b**) at least 41 years old.

▶ Source: GVU's WWW User Surveys™



SOLUTION The number of people surveyed was 1636 + 6617 + 3693 + 491 + 6 = 12,443.

a. Of the people surveyed, 1636 are at most 20 years old. So, the probability is:

$$P(\text{user is at most 20}) = \frac{1636}{12{,}443} \approx 0.131$$

b. Of the people surveyed, 3693 + 491 + 6 = 4190 are at least 41 years old. So, the probability is:

$$P(\text{user is at least 41}) = \frac{4190}{12{,}443} \approx 0.337$$

DED PRACTICE

abulary Check ✓

1. Complete this statement: A probability that involves length, area, or volume is called a(n) _?_ probability.

:oncept Check ✓

2. $P(A) = 0.2$ and $P(B) = 0.6$. Which event is more likely to occur? Explain.

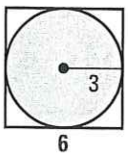
3. Explain the difference between theoretical probability and experimental probability. Give an example of each.

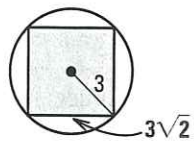
Skill Check ✓

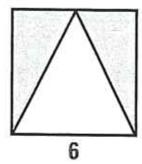
A jar contains 2 red marbles, 3 blue marbles, and 1 green marble. Find the probability of randomly drawing the given type of marble.

4. a red marble
5. a green marble
6. a blue or a green marble
7. a red or a blue marble

Find the probability that a dart thrown at the given target will hit the shaded region. Assume the dart is equally likely to hit any point inside the target. The targets and regions within are either squares, circles, or triangles.

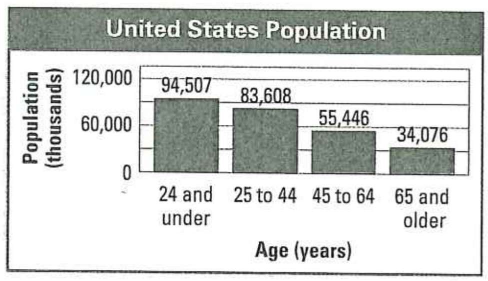
8.


9.


10.


11. **POPULATION** The bar graph shown gives the resident population (in thousands) of the United States in 1997. For a randomly selected person in the United States, find the probability of the given event.

DATA UPDATE of *Statistical Abstract of the United States* data at www.mcdougallittell.com



a. The person is 24 years old or under.
b. The person is at least 45 years old.

CTICE AND APPLICATIONS

DENT HELP

a Practice
lp you master
is on p. 956.

CHOOSING NUMBERS You have an equally likely chance of choosing any integer from 1 through 20. Find the probability of the given event.

12. An odd number is chosen.
13. A number less than 7 is chosen.
14. A perfect square is chosen.
15. A prime number is chosen.
16. A multiple of 3 is chosen.
17. A factor of 240 is chosen.

DENT HELP

k Back
elp with a standard
rd deck, see p. 708.

CHOOSING CARDS A card is drawn randomly from a standard 52-card deck. Find the probability of drawing the given card.

18. the ace of hearts
19. any ace
20. a diamond
21. a red card
22. a card other than 10
23. a face card (a king, queen, or jack)

12.4 Probability of Compound Events

GOAL 1 PROBABILITIES OF UNIONS AND INTERSECTIONS

***What** you should learn*

GOAL 1 Find probabilities of unions and intersections of two events.

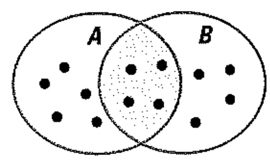
GOAL 2 Use complements to find the probability of an event, as applied in **Example 5**.

***Why** you should learn it*

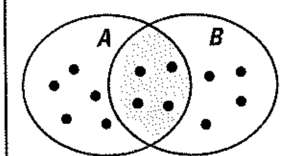
▼ To solve **real-life** problems, such as finding the probability that friends will be in the same college dormitory in **Ex. 49**.

REAL LIFE

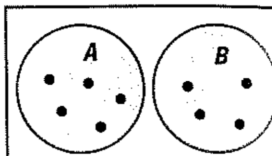
When you consider all the outcomes for either of two events A and B, you form the *union* of A and B. When you consider only the outcomes shared by both A and B, you form the *intersection* of A and B. The union or intersection of two events is called a **compound event**.



Union of A and B



Intersection of A and B



Intersection of A and B is empty.

To find $P(A \text{ or } B)$ you must consider what outcomes, if any, are in the intersection of A and B. If there are none, then A and B are **mutually exclusive events** and $P(A \text{ or } B) = P(A) + P(B)$. If A and B are not mutually exclusive, then the outcomes in the intersection of A and B are counted *twice* when $P(A)$ and $P(B)$ are added. So, $P(A \text{ and } B)$ must be subtracted *once* from the sum.

PROBABILITY OF COMPOUND EVENTS

If A and B are two events, then the probability of A or B is:

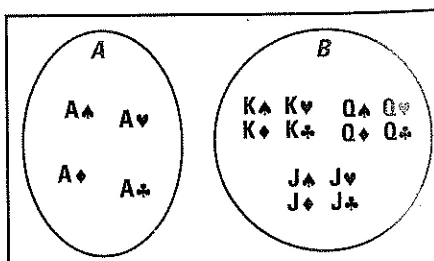
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B are mutually exclusive, then the probability of A or B is:

$$P(A \text{ or } B) = P(A) + P(B)$$

EXAMPLE 1 *Probability of Mutually Exclusive Events*

A card is randomly selected from a standard deck of 52 cards. What is the probability that it is an ace *or* a face card?



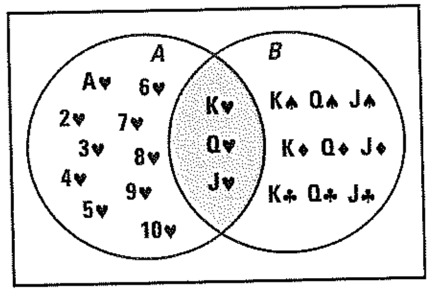
SOLUTION

Let event A be selecting an ace, and let event B be selecting a face card. Event A has 4 outcomes and event B has 12 outcomes. Because A and B are mutually exclusive, the probability is:

$$P(A \text{ or } B) = P(A) + P(B) = \frac{4}{52} + \frac{12}{52} = \frac{16}{52} = \frac{4}{13} \approx 0.308$$

EXAMPLE 2 *Probability of a Compound Event*

A card is randomly selected from a standard deck of 52 cards. What is the probability that the card is a heart *or* a face card?



SOLUTION

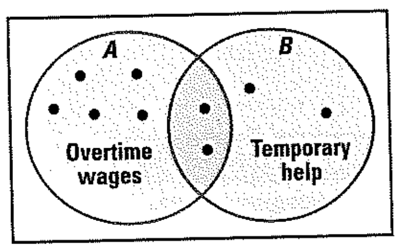
Let event A be selecting a heart, and let event B be selecting a face card. Event A has 13 outcomes and event B has 12 outcomes. Of these, three outcomes are common to A and B. So, the probability of selecting a heart *or* a face card is:

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	**Write general formula.**
$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52}$	**Substitute known probabilities.**
$= \frac{22}{52}$	**Combine terms.**
$= \frac{11}{26}$	**Simplify.**
≈ 0.423	**Use a calculator.**

EXAMPLE 3 *Using Intersection to Find Probability*

REAL LIFE *Business*

Last year a company paid overtime wages *or* hired temporary help during 9 months. Overtime wages were paid during 7 months and temporary help was hired during 4 months. At the end of the year, an auditor examines the accounting records and randomly selects one month to check the company's payroll. What is the probability that the auditor will select a month in which the company paid overtime wages *and* hired temporary help?



SOLUTION

Let event A represent paying overtime wages during a month, and let event B represent hiring temporary help during a month. From the given information you know that:

$$P(A) = \frac{7}{12},\ P(B) = \frac{4}{12}, \text{ and } P(A \text{ or } B) = \frac{9}{12}$$

The probability that the auditor will select a month in which the company paid overtime wages *and* hired temporary help is $P(A \text{ and } B)$.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	**Write general formula.**
$\frac{9}{12} = \frac{7}{12} + \frac{4}{12} - P(A \text{ and } B)$	**Substitute known probabilities.**
$P(A \text{ and } B) = \frac{7}{12} + \frac{4}{12} - \frac{9}{12}$	**Solve for $P(A$ and $B)$.**
$P(A \text{ and } B) = \frac{2}{12} = \frac{1}{6} \approx 0.167$	**Simplify.**

IDED PRACTICE

:abulary Check ✓

Concept Check ✓

1. Describe what it means for two events to be mutually exclusive.

2. Write a formula for computing $P(A \text{ or } B)$ that applies to *any* events A and B. How can you simplify this formula when A and B are mutually exclusive?

3. Are the events A and A' mutually exclusive? Explain.

Skill Check ✓

Events *A* and *B* are mutually exclusive. Find *P*(*A* or *B*).

4. $P(A) = 0.2, P(B) = 0.3$

5. $P(A) = 0.5, P(B) = 0.5$

6. $P(A) = \frac{3}{8}, P(B) = \frac{1}{8}$

7. $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$

Find *P*(*A* or *B*).

8. $P(A) = 0.5, P(B) = 0.4,$ $P(A \text{ and } B) = 0.3$

9. $P(A) = \frac{2}{5}, P(B) = \frac{3}{5},$ $P(A \text{ and } B) = \frac{1}{5}$

Find *P*(*A* and *B*).

10. $P(A) = 0.7, P(B) = 0.2,$ $P(A \text{ or } B) = 0.8$

11. $P(A) = \frac{5}{16}, P(B) = \frac{7}{16},$ $P(A \text{ or } B) = \frac{9}{16}$

Find *P*(*A*′).

12. $P(A) = 0.5$

13. $P(A) = 0.75$

14. $P(A) = \frac{1}{3}$

15. $P(A) = \frac{4}{7}$

ACTICE AND APPLICATIONS

TUDENT HELP

xtra Practice
help you master
kills is on p. 957.

FINDING PROBABILITIES Find the indicated probability. State whether *A* and *B* are mutually exclusive.

16. $P(A) = 0.4$
$P(B) = 0.35$
$P(A \text{ or } B) = 0.5$
$P(A \text{ and } B) = \underline{?}$

17. $P(A) = 0.6$
$P(B) = 0.2$
$P(A \text{ or } B) = \underline{?}$
$P(A \text{ and } B) = 0.1$

18. $P(A) = 0.25$
$P(B) = \underline{?}$
$P(A \text{ or } B) = 0.70$
$P(A \text{ and } B) = 0$

19. $P(A) = \frac{13}{17}$
$P(B) = \underline{?}$
$P(A \text{ or } B) = \frac{14}{17}$
$P(A \text{ and } B) = \frac{6}{17}$

20. $P(A) = \frac{1}{3}$
$P(B) = \frac{1}{4}$
$P(A \text{ or } B) = \frac{7}{12}$
$P(A \text{ and } B) = \underline{?}$

21. $P(A) = \frac{3}{4}$
$P(B) = \frac{1}{3}$
$P(A \text{ or } B) = \underline{?}$
$P(A \text{ and } B) = \frac{1}{4}$

22. $P(A) = 5\%$
$P(B) = 29\%$
$P(A \text{ or } B) = \underline{?}$
$P(A \text{ and } B) = 0\%$

23. $P(A) = 30\%$
$P(B) = \underline{?}$
$P(A \text{ or } B) = 50\%$
$P(A \text{ and } B) = 10\%$

24. $P(A) = 16\%$
$P(B) = 24\%$
$P(A \text{ or } B) = 32\%$
$P(A \text{ and } B) = \underline{?}$

STUDENT HELP

HOMEWORK HELP
:ample 1: Exs. 16–24, 29–34, 42, 43
:ample 2: Exs. 16–24, 29–34, 44, 45
:ample 3: Exs. 16–24, 29–34, 46, 47
:ample 4: Exs. 25–28, 35–40
:ample 5: Exs. 48, 49

FINDING PROBABILITIES OF COMPLEMENTS Find *P*(*A*′).

25. $P(A) = 0.34$

26. $P(A) = 0$

27. $P(A) = \frac{3}{4}$

28. $P(A) = 1$

GOAL 2 USING COMPLEMENTS TO FIND PROBABILITY

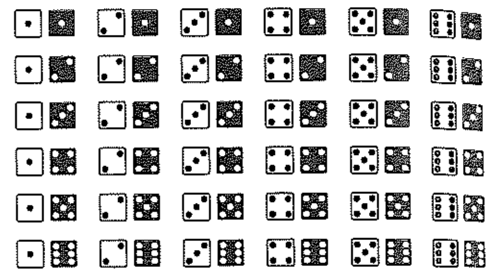
The event A', called the **complement** of event A, consists of all outcomes that are not in A. The notation A' is read as "A prime."

PROBABILITY OF THE COMPLEMENT OF AN EVENT

The probability of the complement of A is $P(A') = 1 - P(A)$.

EXAMPLE 4 *Probabilities of Complements*

When two six-sided dice are tossed, there are 36 possible outcomes as shown. Find the probability of the given event.



a. The sum is not 8.

b. The sum is greater than or equal to 4.

SOLUTION

a. $P(\text{sum is not } 8) = 1 - P(\text{sum is } 8)$

$= 1 - \frac{5}{36}$

$= \frac{31}{36}$

≈ 0.861

b. $P(\text{sum} \geq 4) = 1 - P(\text{sum} < 4)$

$= 1 - \frac{3}{36}$

$= \frac{33}{36}$

$= \frac{11}{12}$

≈ 0.917

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 5 *Using a Complement in Real Life*

HOME ELECTRONICS Four houses in a neighborhood have the same model of garage door opener. Each opener has 4096 possible transmitter codes. What is the probability that at least two of the four houses have the same code?

SOLUTION

The total number of ways to assign codes to the four openers is 4096^4. The number of ways to assign *different* codes to the four openers is $4096 \cdot 4095 \cdot 4094 \cdot 4093$. So, the probability that at least two of the four openers have the same code is:

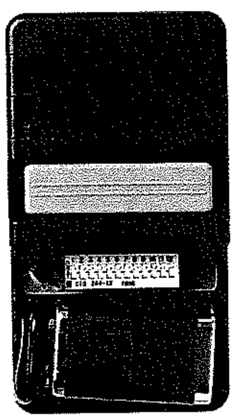
$$P(\text{at least 2 are the same}) = 1 - P(\text{none are the same})$$

$$= 1 - \frac{4096 \cdot 4095 \cdot 4094 \cdot 4093}{4096^4}$$

$$\approx 1 - 0.99854$$

$$= 0.00146$$

FOCUS ON APPLICATIONS



REAL LIFE **HOME ELECTRONICS**
One type of garage door opener has 12 switches that can be set in one of two positions (off or on) to create a code. So for this type of garage door opener, there are 2^{12}, or 4096, possible transmitter codes.

STUDENT HELP

Look Back
For help with a standard 52-card deck in Exs. 29–34, see p. 708.

Look Back
For help with simulations on a graphing calculator in Ex. 41, see p. 723.

CHOOSING CARDS A card is randomly drawn from a standard 52-card deck. Find the probability of the given event. (A face card is a king, queen, or jack.)

29. a queen and a heart
30. a queen or a heart
31. a heart or a diamond
32. a five or a six
33. a five and a six
34. a three or a face card

USING COMPLEMENTS Two six-sided dice are rolled. Find the probability of the given event. (Refer to Example 4 for a diagram of all possible outcomes.)

35. The sum is not 3.
36. The sum is greater than or equal to 5.
37. The sum is neither 3 nor 7.
38. The sum is less than or equal to 10.
39. The sum is greater than 2.
40. The sum is less than 8 or greater than 11.

41. **EXPERIMENTAL PROBABILITIES** Simulate rolling two dice 120 times. Use separate lists for the results of each die, and a third list for the sum. Record the frequency of each sum in a table. Find the experimental probabilities of the events in Exercises 35–40. How do your experimental results compare with the theoretical results?

42. **COMPANY MOVE** An employee of a large national company is promoted to management and will be moved within six months. The employee is told that there is a 33% probability of being moved to Denver, Colorado, and a 50% probability of being moved to Dallas, Texas. What is the probability that the employee will be moved to Dallas or Denver?

43. **CLASS ELECTIONS** You and your best friend are among several candidates running for class president. You estimate that there is a 40% chance you will win the election and a 35% chance your best friend will win. What is the probability that either you or your best friend wins the election?

44. **PARAKEETS** A pet store contains 35 light green parakeets (14 females and 21 males) and 44 sky blue parakeets (28 females and 16 males). You randomly choose one of the parakeets. What is the probability that it is a female or a sky blue parakeet?

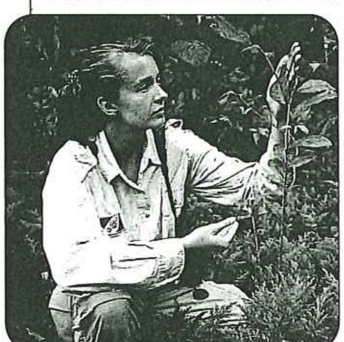
45. **HONORS BANQUET** Of 162 students honored at an academic awards banquet, 48 won awards for mathematics and 78 won awards for English. Fourteen of these students won awards for both mathematics and English. One of the 162 students is chosen at random to be interviewed for a newspaper article. What is the probability that the student interviewed won an award for English or mathematics?

46. **SCIENCE CONNECTION** A tree in a forest is not growing properly. A botanist determines that there is an 85% probability the tree has a disease or is being damaged by insects, a 45% probability it has a disease, and a 50% probability it is being damaged by insects. What is the probability that the tree both has a disease and is being damaged by insects?

47. **RAIN** A weather forecaster says that the probability it will rain on Saturday or Sunday is 50%, the probability it will rain on Saturday is 20%, and the probability it will rain on Sunday is 40%. What is the probability that it will rain on both Saturday and Sunday?

48. **POTLUCK DINNER** The organizer of a potluck dinner sends 5 people a list of 8 different recipes and asks each person to bring one of the items on the list. If all 5 people randomly choose a recipe from the list, what is the probability that at least 2 will bring the same thing?

FOCUS ON CAREERS



REAL LIFE **BOTANIST**
A botanist studies plants and their environment. Some botanists specialize in the causes and cures of plant illnesses, as discussed in Ex. 46.

INTERNET CAREER LINK
www.mcdougallittell.com

12.5 Probability of Independent and Dependent Events

What you should learn

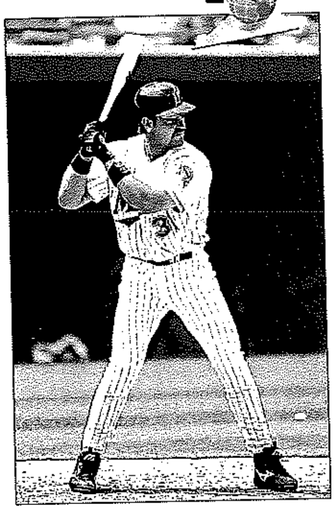
GOAL 1 Find the probability of independent events.

GOAL 2 Find the probability of dependent events, as applied in **Ex. 33**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the probability that the Florida Marlins win three games in a row in **Example 2**.

REAL LIFE



GOAL 1 PROBABILITIES OF INDEPENDENT EVENTS

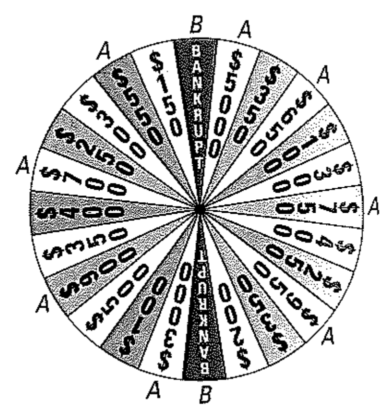
Two events are **independent** if the occurrence of one has no effect on the occurrence of the other. For instance, if a coin is tossed twice, the outcome of the first toss (heads or tails) has no effect on the outcome of the second toss.

PROBABILITY OF INDEPENDENT EVENTS

If A and B are independent events, then the probability that both A and B occur is $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE 1 *Probability of Two Independent Events*

You are playing a game that involves spinning the money wheel shown. During your turn you get to spin the wheel twice. What is the probability that you get more than \$500 on your first spin and then go bankrupt on your second spin?



SOLUTION Let event A be getting more than \$500 on the first spin, and let event B be going bankrupt on the second spin. The two events are independent. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{8}{24} \cdot \frac{2}{24} = \frac{1}{36} \approx 0.028$$

The formula given above for the probability of two independent events can be extended to the probability of three or more independent events.

EXAMPLE 2 *Probability of Three Independent Events*

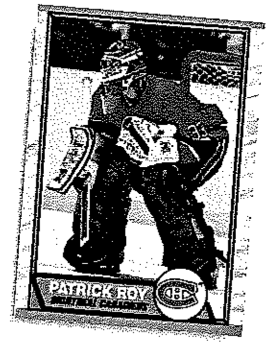
BASEBALL During the 1997 baseball season, the Florida Marlins won 5 out of 7 home games and 3 out of 7 away games against the San Francisco Giants. During the 1997 National League Division Series with the Giants, the Marlins played the first two games at home and the third game away. The Marlins won all three games. Estimate the probability of this happening. ▶ Source: The Florida Marlins

SOLUTION Let events A, B, and C be winning the first, second, and third games. The three events are independent and have experimental probabilities based on the regular season games. So, the probability of winning the first three games is:

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C) = \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{3}{7} = \frac{75}{343} \approx 0.219$$

Real Life
Trading Cards

EXAMPLE 3 *Using a Complement to Find a Probability*



You collect hockey trading cards. For one team there are 25 different cards in the set, and you have all of them except for the starting goalie card. To try and get this card, you buy 8 packs of 5 cards each. All cards in a pack are different and each of the cards is equally likely to be in a given pack. Find the probability that you will get at least one starting goalie card.

SOLUTION

In one pack the probability of *not* getting the starting goalie card is:

$$P(\text{no starting goalie}) = \frac{{}_{24}C_5}{{}_{25}C_5}$$

Buying packs of cards are independent events, so the probability of getting at least one starting goalie card in the 8 packs is:

$$\begin{aligned} P(\text{at least one starting goalie}) &= 1 - P(\text{no starting goalie in any pack}) \\ &= 1 - \left(\frac{{}_{24}C_5}{{}_{25}C_5}\right)^8 \\ &\approx 0.832 \end{aligned}$$

Real Life
Manufacturing

EXAMPLE 4 *Solving a Probability Equation*

A computer chip manufacturer has found that only 1 out of 1000 of its chips is defective. You are ordering a shipment of chips for the computer store where you work. How many chips can you order before the probability that at least one chip is defective reaches 50%?

SOLUTION

STUDENT HELP

Look Back
For help with solving exponential equations, see p. 501.

Let n be the number of chips you order. From the given information you know that $P(\text{chip is } \textit{not} \text{ defective}) = \frac{999}{1000} = 0.999$. Use this probability and the fact that each chip ordered represents an independent event to find the value of n.

$P(\text{at least one chip is defective}) = 0.5$	**Write given assumption.**
$1 - P(\text{no chips are defective}) = 0.5$	**Use complement.**
$1 - (0.999)^n = 0.5$	**Substitute known probability.**
$-(0.999)^n = -0.5$	**Subtract 1 from each side.**
$(0.999)^n = 0.5$	**Divide each side by –1.**
$n = \frac{\log 0.5}{\log 0.999}$	**Solve for *n*.**
$n \approx 693$	**Use a calculator.**

▶ If you order 693 chips, you have a 50% chance of getting a defective chip. Therefore, you can order 692 chips before the probability that at least one chip is defective reaches 50%.

GOAL 2 PROBABILITIES OF DEPENDENT EVENTS

Two events A and B are **dependent events** if the occurrence of one affects the occurrence of the other. The probability that B will occur given that A has occurred is called the **conditional probability** of B given A and is written $P(B \mid A)$.

PROBABILITY OF DEPENDENT EVENTS

If A and B are dependent events, then the probability that both A and B occur is $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

REAL LIFE

Endangered Species

EXAMPLE 5 *Finding Conditional Probabilities*

The table shows the number of endangered and threatened animal species in the United States as of November 30, 1998. Find (**a**) the probability that a listed animal is a reptile and (**b**) the probability that an endangered animal is a reptile.

▶ Source: United States Fish and Wildlife Service

	Mammals	Birds	Reptiles	Amphibians	Other
Endangered	59	75	14	9	198
Threatened	8	15	21	7	69

SOLUTION

a. $P(\text{reptile}) = \dfrac{\text{number of reptiles}}{\text{total number of animals}} = \dfrac{35}{475} \approx 0.0737$

b. $P(\text{reptile} \mid \text{endangered}) = \dfrac{\text{number of endangered reptiles}}{\text{total number of endangered animals}}$

$= \dfrac{14}{355} \approx 0.0394$

STUDENT HELP

Look Back
For help with a standard 52-card deck, see p. 708.

EXAMPLE 6 *Comparing Dependent and Independent Events*

You randomly select two cards from a standard 52-card deck. What is the probability that the first card is not a face card (a king, queen, or jack) and the second card is a face card if (**a**) you replace the first card before selecting the second, and (**b**) you do *not* replace the first card?

SOLUTION

a. If you replace the first card before selecting the second card, then A and B are independent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{40}{52} \cdot \frac{12}{52} = \frac{30}{169} \approx 0.178$$

b. If you do *not* replace the first card before selecting the second card, then A and B are dependent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = \frac{40}{52} \cdot \frac{12}{51} = \frac{40}{221} \approx 0.181$$

The formula for finding probabilities of dependent events can be extended to three or more events, as shown in Example 7.

REAL LIFE
Dining Out

EXAMPLE 7 *Probability of Three Dependent Events*

You and two friends go to a restaurant and order a sandwich. The menu has 10 types of sandwiches and each of you is equally likely to order any type. What is the probability that each of you orders a different type?

SOLUTION

Let event A be that you order a sandwich, event B be that one friend orders a different type, and event C be that your other friend orders a third type. These events are dependent. So, the probability that each of you orders a different type is:

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } B)$$

$$= \frac{10}{10} \cdot \frac{9}{10} \cdot \frac{8}{10}$$

$$= \frac{18}{25} = 0.72$$

STUDENT HELP

Study Tip

You can also use the fundamental counting principle to find the probability in Example 7.

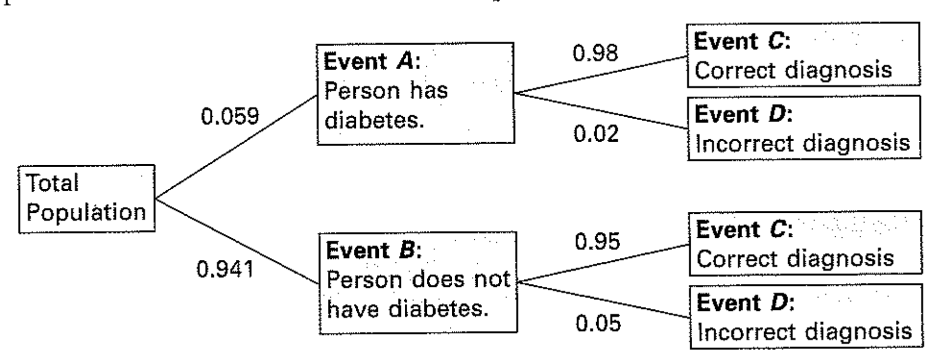
$$P(\text{all different}) = \frac{\text{no. of different orders}}{\text{no. of possible orders}} = \frac{10 \cdot 9 \cdot 8}{10 \cdot 10 \cdot 10} = 0.72$$

EXAMPLE 8 *Using a Tree Diagram to Find Conditional Probabilities*

HEALTH The American Diabetes Association estimates that 5.9% of Americans have diabetes. Suppose that a medical lab has developed a simple diagnostic test for diabetes that is 98% accurate for people who have the disease and 95% accurate for people who do not have it. If the medical lab gives the test to a randomly selected person, what is the probability that the diagnosis is correct?

SOLUTION

A probability tree diagram, where the probabilities are given along the branches, can help you see the different ways to obtain a correct diagnosis. Notice that the probabilities for all branches from the same point must sum to 1.



So, the probability that the diagnosis is correct is:

$P(C) = P(A \text{ and } C) + P(B \text{ and } C)$	**Follow branches leading to *C*.**
$= P(A) \cdot P(C \mid A) + P(B) \cdot P(C \mid B)$	**Use formula for dependent events.**
$= (0.059)(0.98) + (0.941)(0.95)$	**Substitute.**
≈ 0.952	**Use a calculator.**

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain the difference between dependent events and independent events, and give an example of each.

Concept Check ✓

2. If event A is drawing a queen from a deck of cards and event B is drawing a king from the remaining cards, are events A and B dependent or independent?
3. If event A is rolling a two on a six-sided die and event B is rolling a four on a different six-sided die, are events A and B dependent or independent?

Skill Check ✓

Events *A* and *B* are independent. Find the indicated probability.

4. $P(A) = 0.3$
   $P(B) = 0.9$
   $P(A \text{ and } B) = \underline{?}$
5. $P(A) = \underline{?}$
   $P(B) = 0.3$
   $P(A \text{ and } B) = 0.06$
6. $P(A) = 0.75$
   $P(B) = \underline{?}$
   $P(A \text{ and } B) = 0.15$

Events *A* and *B* are dependent. Find the indicated probability.

7. $P(A) = 0.1$
   $P(B \mid A) = 0.8$
   $P(A \text{ and } B) = \underline{?}$
8. $P(A) = \underline{?}$
   $P(B \mid A) = 0.5$
   $P(A \text{ and } B) = 0.25$
9. $P(A) = 0.9$
   $P(B \mid A) = \underline{?}$
   $P(A \text{ and } B) = 0.54$

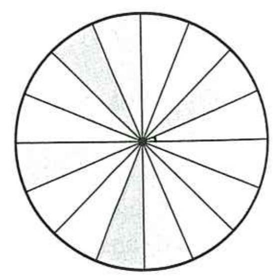
READING LIST In Exercises 10 and 11, use the following information.
Three friends are taking an English class that has a summer reading list. Each student is required to read one book from the list, which contains 3 biographies, 10 classics, and 5 historical novels.

10. Find the probability that the first friend chooses a biography, the second friend chooses a classic, and the third friend chooses a historical novel.
11. Find the probability that the three friends each choose a different classic.

PRACTICE AND APPLICATIONS

STUDENT HELP
Extra Practice to help you master skills is on p. 957.

SPINNING A WHEEL You are playing a game that involves spinning the wheel shown. Find the probability of spinning the given colors.



12. red, then blue
13. red, then green
14. yellow, then red
15. green, then yellow
16. blue, then yellow, then green
17. green, then red, then blue

STUDENT HELP
HOMEWORK HELP
Example 1: Exs. 12–15
Example 2: Exs. 16, 17, 24, 25
Example 3: Exs. 26, 27
Example 4: Exs. 28, 29
Example 5: Exs. 30–32
Example 6: Exs. 18–21
Example 7: Exs. 22, 23, 33, 34
Example 8: Exs. 35, 36

DRAWING CARDS Find the probability of drawing the given cards from a standard 52-card deck (a) with replacement and (b) without replacement.

18. a heart, then a diamond
19. a jack, then a king
20. a 2, then a face card (K, Q, or J)
21. a face card (K, Q, or J), then a 2
22. an ace, then a 2, then a 3
23. a heart, then a diamond, then another heart
24. **GAMES** You are playing a game that involves drawing three numbers from a hat. There are 25 pieces of paper numbered 1 to 25 in the hat. Each number is replaced after it is drawn. What is the probability that each number is greater than 20 or less than 4?

25. **LAWN CARE** The owner of a one-man lawn mowing business owns three old and unreliable riding mowers. As long as one of the three is working he can stay productive. From past experience, one of the mowers is unusable 12 percent of the time, one 6 percent of the time, and one 20 percent of the time. Find the probability that all three mowers are unusable on a given day.

26. **TRADING CARDS** You collect movie trading cards, which have different scenes from a movie. For one movie there are 90 different cards in the set, and you have all of them except the final scene. To try and get this card, you buy 10 packs of 8 cards each. All cards in a pack are different and each of the cards is equally likely to be in a given pack. Find the probability that you will get the final scene.

27. **FREE THROWS** Chris Mullin of the Indiana Pacers led the National Basketball Association in free-throw percentage during the 1997–1998 season. He made 93.9% of his free-throw attempts. If he attempted 10 free throws in a game, what is the probability that he missed at least one? ▶ Source: NBA

28. **MANUFACTURING** Look back at Example 4. Suppose the computer chip manufacturer has improved quality control so that only 1 out of 10,000 of its chips is defective. Now how many chips can you order before the probability that at least one chip is defective reaches 50%?

29. **LOTTERY** To win a state lottery, a player must correctly match six different numbers from 1 to 42. If a computer randomly assigns six numbers per ticket, how many tickets would a person have to buy to have a 1% chance of winning?

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 29.

STATISTICS CONNECTION **In Exercises 30 and 31, use the following information.** The table, based on a Gallup Poll, shows the number of voters (in 1000's) by party affiliation who were expected to vote for Bill Clinton and Bob Dole in the 1996 Presidential election. ▶ Source: The Gallup Organization

	Democrat	Republican	Independent
Clinton	31,378	3,340	12,685
Dole	2,092	28,386	8,721

30. Find the probability that a randomly selected person voted for Clinton.

31. Find the probability that a randomly selected Democrat voted for Clinton.

32. **TEACHERS** In the United States during the 1993–1994 school year, 39.6% of all male teachers and 26.1% of all female teachers had twenty years or more of full-time teaching experience. That year 694,000 males and 1,867,000 females were teachers. What is the probability that a randomly chosen teacher in the United States that year was a female with twenty years or more of full-time teaching experience?

DATA UPDATE of *Statistical Abstract of the United States* data at www.mcdougallittell.com

FOCUS ON CAREERS

TEACHER In addition to teaching, teachers plan daily lessons and activities, assign and correct homework, and prepare and grade exams. They also monitor homerooms, study halls, and cafeterias, meet with parents, and supervise extracurricular activities.

CAREER LINK www.mcdougallittell.com

33. **COSTUMES** You and four of your friends go to the same store at different times to buy costumes for a costume party. There are 20 different costumes at the store, and the store has at least five duplicates of each costume. Find the probability that all five of you choose different costumes.

34. **AIRPLANE MEALS** On a long flight an airline usually serves a meal. If there are 2 choices for the meal, what is the probability that all 6 people in the first row choose the same meal assuming choices are made independently?

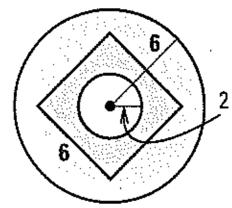
IZ 2

Self-Test for Lessons 12.3–12.5

A jar contains 6 blue marbles, 12 green marbles, and 7 yellow marbles. Find the probability of randomly drawing the given marble. (Lesson 12.3)

1. a green marble **2.** a blue marble **3.** a green or a blue marble

Find the probability that a dart thrown at the circular target shown will hit the given region. Assume the dart is equally likely to hit any point inside the target. (Lesson 12.3)



4. the center circle **5.** outside the square

6. the area inside the square but outside the center circle

Find the indicated probability. (Lesson 12.4)

7. $P(A) = 0.7$
$P(B) = 0.2$
$P(A \text{ or } B) = \underline{?}$
$P(A \text{ and } B) = 0.1$

8. $P(A) = 0.5$
$P(B) = 0.4$
$P(A \text{ or } B) = 0.9$
$P(A \text{ and } B) = \underline{?}$

9. $P(A) = 0.25$
$P(A') = \underline{?}$

10. **FRUIT** You and four friends are in line at lunch and are each selecting a piece of fruit to eat. If there are 5 types of fruit available, what is the probability that you each select a different type? **(Lesson 12.5)**

ATH & istory

Probability Theory

APPLICATION LINK
www.mcdougallittell.com

EN

IN 1654 Blaise Pascal and Pierre Fermat solved the first probability problem, which asked how to divide the stakes of an interrupted game of chance between two players of equal ability. Suppose Player A has 2 points, Player B has 1 point, and the game is won by the first player to score 4 out of a possible 7 points.

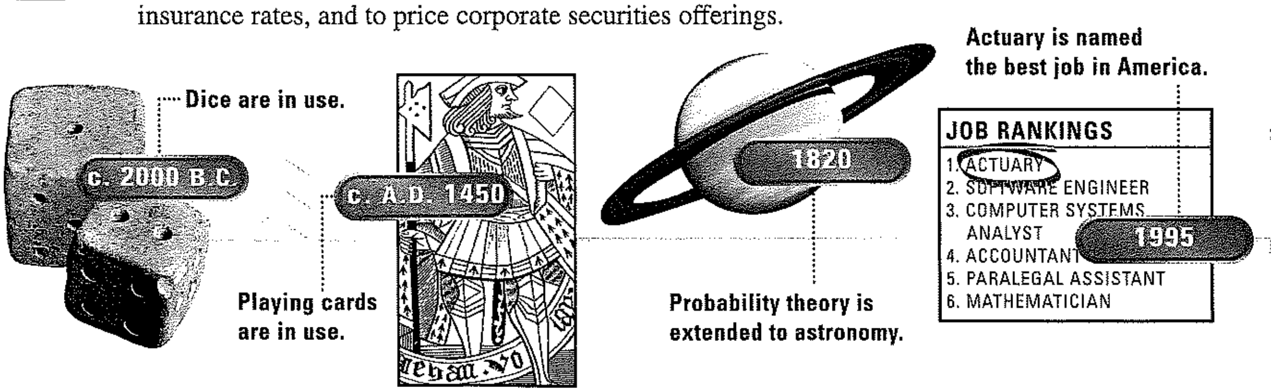
1. If the stakes are divided based on which player is closest to winning, what fraction of the stakes should each player receive?

2. If the stakes are divided based on the number of points each player has so far, what fraction of the stakes should each player receive?

3. What is the probability Player A will win the game? What is the probability Player B will win the game? Based on these probabilities, what fraction of the stakes should each player receive?

OW

TODAY probability theory is used to decide more than just games of chance. Actuaries, for example, use probability theory to design financial plans, to calculate insurance rates, and to price corporate securities offerings.



12.6 Binomial Distributions

hat you should learn

1 Find binomial
babilities and analyze
omial distributions.

2 Test a hypothesis,
applied in **Example 4**.

hy you should learn it

To solve **real-life** prob-
s, such as determining
ether a computer manu-
turer's claim is correct
x. 46.

REAL LIFE

GOAL 1 FINDING BINOMIAL PROBABILITIES

There are many probability experiments in which the results of each trial can be reduced to two outcomes. If such an experiment satisfies the following conditions, then it is called a **binomial experiment**.

- There are n independent trials.
- Each trial has only two possible outcomes: success and failure.
- The probability of success is the same for each trial. This probability is denoted by p. The probability of failure is given by $1 - p$.

FINDING A BINOMIAL PROBABILITY

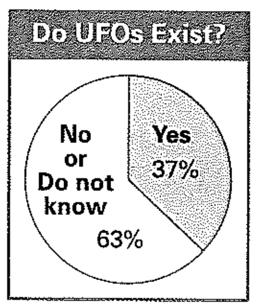
For a binomial experiment consisting of n trials, the probability of exactly k successes is

$$P(k \text{ successes}) = {}_nC_k p^k (1 - p)^{n - k}$$

where the probability of success on each trial is p.

EXAMPLE 1 *Finding a Binomial Probability*

UFOs According to a survey taken by *USA Today*, about 37% of adults believe that Unidentified Flying Objects (UFOs) really exist. Suppose you randomly survey 6 adults. What is the probability that exactly 2 of them believe that UFOs really exist?



SOLUTION

Let $p = 0.37$ be the probability that a randomly selected adult believes that UFOs really exist. By surveying 6 adults, you are conducting $n = 6$ independent trials. The probability of getting exactly $k = 2$ successes is:

$$P(k = 2) = {}_6C_2(0.37)^2(1 - 0.37)^{6-2}$$

$$= \frac{6!}{4! \cdot 2!}(0.37)^2(0.63)^4$$

$$\approx 0.323$$

▶ The probability that exactly 2 of the people surveyed believe that UFOs really exist is about 32%.

....

A **binomial distribution** shows the probabilities of all possible numbers of successes in a binomial experiment, as illustrated in Example 2 on the next page.

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain the difference between a binomial experiment and a binomial distribution.

Concept Check ✓

2. You draw ten cards from a standard 52-card deck without replacement. Is this a binomial experiment? Explain.

3. Consider the binomial distribution shown. Is the distribution skewed or symmetric? Explain.

k	$P(k)$
0	0.0625
1	0.25
2	0.375
3	0.25
4	0.0625

Ex. 3

Skill Check ✓

Calculate the probability of k successes for a binomial experiment consisting of n trials with probability p of success on each trial.

4. $k = 7, n = 12, p = 0.7$ **5.** $k \leq 3, n = 14, p = 0.45$

A binomial experiment consists of n trials with probability p of success on each trial. Draw a histogram of the binomial distribution that shows the probability of exactly k successes. Then find the most likely number of successes.

6. $n = 6, p = 0.5$ **7.** $n = 8, p = 0.33$ **8.** $n = 10, p = 0.25$

9. **CLASS RINGS** You read an article that claims only 30% of graduating seniors will buy a class ring. To test this claim you survey 15 randomly selected seniors in your school and find that 4 are planning to buy class rings. Should you reject the claim? Explain. ▶ Source: *America by the Numbers*

PRACTICE AND APPLICATIONS

STUDENT HELP

▶ **Extra Practice** to help you master skills is on p. 957.

CALCULATING PROBABILITIES **Calculate the probability of tossing a coin 20 times and getting the given number of heads.**

10. 1 **11.** 3 **12.** 5 **13.** 9

14. 10 **15.** 11 **16.** 15 **17.** 17

BINOMIAL PROBABILITIES **Calculate the probability of randomly guessing the given number of correct answers on a 30-question multiple-choice exam that has choices A, B, C, and D for each question.**

18. 0 **19.** 2 **20.** 5 **21.** 10

22. 15 **23.** 20 **24.** 25 **25.** 30

BINOMIAL DISTRIBUTIONS **Calculate the probability of k successes for a binomial experiment consisting of n trials with probability p of success on each trial.**

26. $k \geq 3, n = 5, p = 0.2$ **27.** $k \leq 2, n = 6, p = 0.5$

28. $k \leq 1, n = 9, p = 0.15$ **29.** $k \leq 5, n = 12, p = 0.64$

STUDENT HELP

▶ **HOMEWORK HELP**
Example 1: Exs. 10–25, 36, 37
Example 2: Exs. 26–29, 38–41
Example 3: Exs. 30–35, 42–45
Example 4: Exs. 46–48

HISTOGRAMS **A binomial experiment consists of n trials with probability p of success on each trial. Draw a histogram of the binomial distribution that shows the probability of exactly k successes. Then find the most likely number of successes.**

30. $n = 2, p = 0.4$ **31.** $n = 4, p = 0.7$ **32.** $n = 5, p = 0.17$

33. $n = 8, p = 0.92$ **34.** $n = 9, p = 0.125$ **35.** $n = 12, p = 0.033$