Part 2:

PROBABILITY PRACTICE

12.1 The Fundamental Counting Principle and Permutations

Vhat you should learn

OAL 1 Use the funda-
iental counting principle to
ount the number of ways an
vent can happen.

OAL 2 Use permutations
) count the number of ways
n event can happen, as
pplied in **Ex. 62.**

Vhy you should learn it

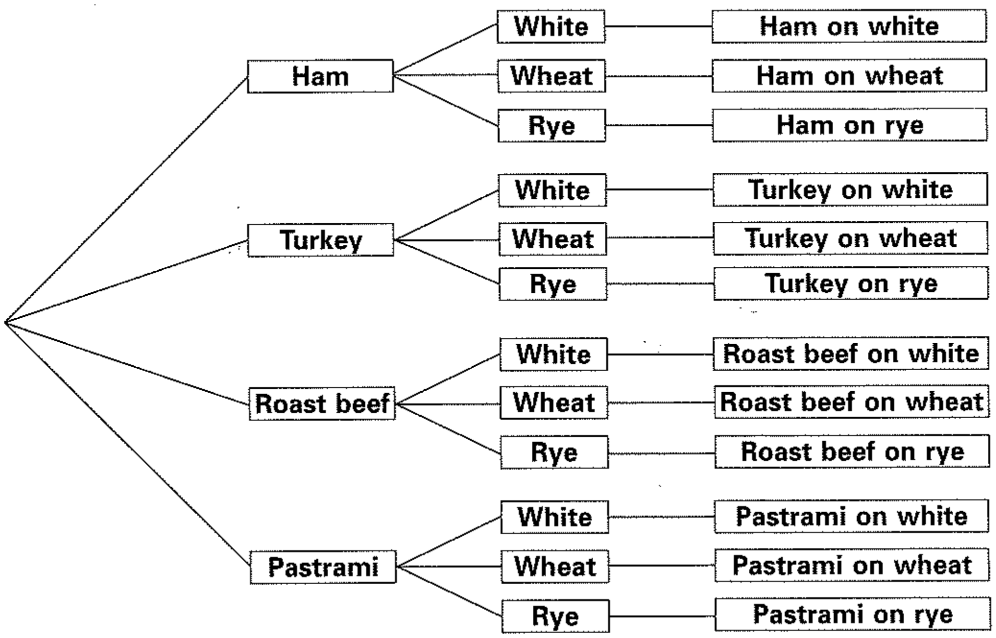
▼ To find the number of
vays a **real-life** event can
appen, such as the number
f ways skiers can finish in
n aerial competition
n **Example 3.**

REAL LIFE

GOAL 1 THE FUNDAMENTAL COUNTING PRINCIPLE

In many real-life problems you want to count the number of possibilities. For instance, suppose you own a small deli. You offer 4 types of meat (ham, turkey, roast beef, and pastrami) and 3 types of bread (white, wheat, and rye). How many choices do your customers have for a meat sandwich?

One way to answer this question is to use a *tree diagram*, as shown below. From the list on the right you can see that there are 12 choices.



Another way to count the number of possible sandwiches is to use the *fundamental counting principle*. Because you have 4 choices for meat and 3 choices for bread, the total number of choices is $4 \cdot 3 = 12$.

FUNDAMENTAL COUNTING PRINCIPLE

TWO EVENTS If one event can occur in m ways and another event can occur in n ways, then the number of ways that *both* events can occur is $m \cdot n$.

For instance, if one event can occur in 2 ways and another event can occur in 5 ways, then both events can occur in $2 \cdot 5 = 10$ ways.

THREE OR MORE EVENTS The fundamental counting principle can be extended to three or more events. For example, if three events can occur in m, n, and p ways, then the number of ways that *all* three events can occur is $m \cdot n \cdot p$.

For instance, if three events can occur in 2, 5, and 7 ways, then all three events can occur in $2 \cdot 5 \cdot 7 = 70$ ways.

FOCUS ON CAREERS

POLICE DETECTIVE

A police detective is an officer who collects facts and evidence for criminal cases. Part of a detective's duties may include helping witnesses identify suspects.

CAREER LINK
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EXAMPLE 1 *Using the Fundamental Counting Principle*

CRIMINOLOGY Police use photographs of various facial features to help witnesses identify suspects. One basic identification kit contains 195 hairlines, 99 eyes and eyebrows, 89 noses, 105 mouths, and 74 chins and cheeks.

▶ Source: *Readers' Digest: How In The World?*

a. The developer of the identification kit claims that it can produce billions of different faces. Is this claim correct?

b. A witness can clearly remember the hairline and the eyes and eyebrows of a suspect. How many different faces can be produced with this information?

SOLUTION

a. You can use the fundamental counting principle to find the total number of different faces.

Number of faces $= 195 \cdot 99 \cdot 89 \cdot 105 \cdot 74 = 13{,}349{,}986{,}650$

▶ The developer's claim is correct since the kit can produce over 13 billion faces.

b. Because the witness clearly remembers the hairline and the eyes and eyebrows, there is only 1 choice for each of these features. You can use the fundamental counting principle to find the number of different faces.

Number of faces $= 1 \cdot 1 \cdot 89 \cdot 105 \cdot 74 = 691{,}530$

▶ The number of faces that can be produced has been reduced to 691,530.

REAL LIFE *License Plates*

EXAMPLE 2 *Using the Fundamental Counting Principle with Repetition*

The standard configuration for a New York license plate is 3 digits followed by 3 letters.

▶ Source: New York State Department of Motor Vehicles

a. How many different license plates are possible if digits and letters can be repeated?

b. How many different license plates are possible if digits and letters cannot be repeated?

SOLUTION

a. There are 10 choices for each digit and 26 choices for each letter. You can use the fundamental counting principle to find the number of different plates.

Number of plates $= 10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 = 17{,}576{,}000$

▶ The number of different license plates is 17,576,000.

b. If you cannot repeat digits there are still 10 choices for the first digit, but then only 9 remaining choices for the second digit and only 8 remaining choices for the third digit. Similarly, there are 26 choices for the first letter, 25 choices for the second letter, and 24 choices for the third letter. You can use the fundamental counting principle to find the number of different plates.

Number of plates $= 10 \cdot 9 \cdot 8 \cdot 26 \cdot 25 \cdot 24 = 11{,}232{,}000$

▶ The number of different license plates is 11,232,000.

GOAL 2 USING PERMUTATIONS

> **STUDENT HELP**
>
> **Study Tip**
> Recall from Lesson 11.5 that $n!$ is read as "n factorial." Also note that $0! = 1$ and $1! = 1$.

An ordering of n objects is a **permutation** of the objects. For instance, there are six permutations of the letters A, B, and C: ABC, ACB, BAC, BCA, CAB, CBA.

The fundamental counting principle can be used to determine the number of permutations of n objects. For instance, you can find the number of ways you can arrange the letters A, B, and C by multiplying. There are 3 choices for the first letter, 2 choices for the second letter, and 1 choice for the third letter, so there are $3 \cdot 2 \cdot 1 = 6$ ways to arrange the letters.

In general, the number of permutations of n distinct objects is:

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$$

EXAMPLE 3 *Finding the Number of Permutations*

REAL LIFE Sports

Twelve skiers are competing in the final round of the Olympic freestyle skiing aerial competition.

a. In how many different ways can the skiers finish the competition? (Assume there are no ties.)

b. In how many different ways can 3 of the skiers finish first, second, and third to win the gold, silver, and bronze medals?

SOLUTION

a. There are 12! different ways that the skiers can finish the competition.

$$12! = 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 479{,}001{,}600$$

b. Any of the 12 skiers can finish first, then any of the remaining 11 skiers can finish second, and finally any of the remaining 10 skiers can finish third. So, the number of ways that the skiers can win the medals is:

$$12 \cdot 11 \cdot 10 = 1320$$

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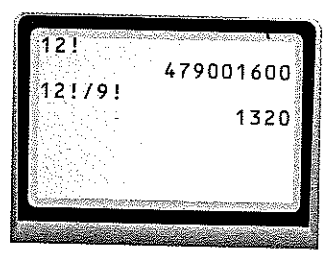
> **STUDENT HELP**
>
> INTERNET **KEYSTROKE HELP**
> Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

Some calculators have special keys to evaluate factorials. The solution to Example 3 is shown.



The number in part (b) of Example 3 is called the number of permutations of 12 objects taken 3 at a time, is denoted by $_{12}P_3$, and is given by $\frac{12!}{(12-3)!}$.

> **STUDENT HELP**
>
> **Derivations**
> For a derivation of the formula for the permutation of n objects taken r at a time, see p. 899.

PERMUTATIONS OF n OBJECTS TAKEN r AT A TIME

The number of permutations of r objects taken from a group of n distinct objects is denoted by $_nP_r$ and is given by:

$$_nP_r = \frac{n!}{(n-r)!}$$

REAL LIFE
College Visits

EXAMPLE 4 *Finding Permutations of n Objects Taken r at a Time*

You are considering 10 different colleges. Before you decide to apply to the colleges, you want to visit some or all of them. In how many orders can you visit (**a**) 6 of the colleges and (**b**) all 10 colleges?

SOLUTION

a. The number of permutations of 10 objects taken 6 at a time is:

$$_{10}P_6 = \frac{10!}{(10-6)!} = \frac{10!}{4!} = \frac{3,628,800}{24} = 151,200$$

b. The number of permutations of 10 objects taken 10 at a time is:

$$_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!} = 10! = 3,628,800$$

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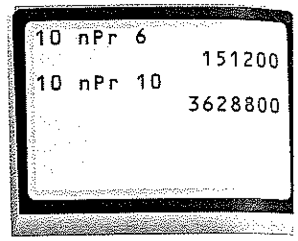
STUDENT HELP

KEYSTROKE HELP Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

Some calculators have special keys that are programmed to evaluate $_nP_r$. The solution to Example 4 is shown.



So far you have been finding permutations of *distinct* objects. If some of the objects are repeated, then some of the permutations are not distinguishable. For instance, of the six ways to order the letters M, O, and M—

MOM OMM MMO

MOM OMM MMO

—only three are distinguishable without color: MOM, OMM, and MMO. In this case, the number of permutations is $\frac{3!}{2!} = \frac{6}{2} = 3$, not $3! = 6$.

PERMUTATIONS WITH REPETITION

The number of distinguishable permutations of n objects where one object is repeated q_1 times, another is repeated q_2 times, and so on is:

$$\frac{n!}{q_1! \cdot q_2! \cdot \ldots \cdot q_k!}$$

EXAMPLE 5 *Finding Permutations with Repetition*

Find the number of distinguishable permutations of the letters in (**a**) OHIO and (**b**) MISSISSIPPI.

SOLUTION

a. OHIO has 4 letters of which O is repeated 2 times. So, the number of distinguishable permutations is $\frac{4!}{2!} = \frac{24}{2} = 12$.

b. MISSISSIPPI has 11 letters of which I is repeated 4 times, S is repeated 4 times, and P is repeated 2 times. So, the number of distinguishable permutations is $\frac{11!}{4! \cdot 4! \cdot 2!} = \frac{39,916,800}{24 \cdot 24 \cdot 2} = 34,650$.

GUIDED PRACTICE

Vocabulary Check ✓

1. What is a permutation of n objects?

Concept Check ✓

2. Explain how the fundamental counting principle can be used to justify the formula for the number of permutations of n distinct objects.

3. Rita found the number of distinguishable permutations of the letters in OHIO by evaluating the expression $\frac{4!}{2! \cdot 1! \cdot 1!}$. Does this method give the same answer as in part (a) of Example 5? Explain.

4. **ERROR ANALYSIS** Explain the error in calculating how many three-digit numbers from 000 to 999 have only even digits.

Number of 3-digit numbers with only even digits
$= 5 \cdot 4 \cdot 3$
$= 60$

Skill Check ✓

Find the number of permutations of n distinct objects.

5. $n = 2$ **6.** $n = 6$ **7.** $n = 1$ **8.** $n = 4$

Find the number of permutations of n objects taken r at a time.

9. $n = 6, r = 3$ **10.** $n = 5, r = 1$ **11.** $n = 3, r = 3$ **12.** $n = 10, r = 2$

Find the number of permutations of n objects where one or more objects are repeated the given number of times.

13. 7 objects with one object repeated 4 times

14. 5 objects with one object repeated 3 times and a second object repeated 2 times

PRACTICE AND APPLICATIONS

STUDENT HELP

→ **Extra Practice** to help you master skills is on p. 956.

STUDENT HELP

→ HOMEWORK HELP

Example 1: Exs. 15–18, 55, 56
Example 2: Exs. 19–22, 57
Example 3: Exs. 23–30, 39–46, 59, 60
Example 4: Exs. 31–38, 61
Example 5: Exs. 47–54, 62, 63

FUNDAMENTAL COUNTING PRINCIPLE **Each event can occur in the given number of ways. Find the number of ways all of the events can occur.**

15. Event 1: 1 way, Event 2: 3 ways

16. Event 1: 3 ways, Event 2: 5 ways

17. Event 1: 2 ways, Event 2: 4 ways, Event 3: 5 ways

18. Event 1: 4 ways, Event 2: 6 ways, Event 3: 9 ways, Event 4: 7 ways

LICENSE PLATES **For the given configuration, determine how many different license plates are possible if (a) digits and letters can be repeated, and (b) digits and letters cannot be repeated.**

19. 3 letters followed by 3 digits

20. 2 digits followed by 4 letters

21. 4 digits followed by 2 letters

22. 5 letters followed by 1 digit

FACTORIALS **Evaluate the factorial.**

23. 8! **24.** 5! **25.** 10! **26.** 9!

27. 0! **28.** 7! **29.** 3! **30.** 12!

PERMUTATIONS **Find the number of permutations.**

31. ${}_3P_3$ **32.** ${}_5P_2$ **33.** ${}_2P_1$ **34.** ${}_7P_6$

35. ${}_8P_5$ **36.** ${}_9P_4$ **37.** ${}_{12}P_3$ **38.** ${}_{16}P_0$

FOCUS ON APPLICATIONS

REAL LIFE **COMPUTER SECURITY** On the Internet there are three main ways to secure a site: restrict which addresses can access the site, use public key cryptography, or require a user name and password.

PERMUTATIONS WITHOUT REPETITION **Find the number of distinguishable permutations of the letters in the word.**

39. HI **40.** JET **41.** IOWA **42.** TEXAS

43. PENCIL **44.** FLORIDA **45.** MAGNETIC **46.** GOLDFINCH

PERMUTATIONS WITH REPETITION **Find the number of distinguishable permutations of the letters in the word.**

47. DAD **48.** PUPPY **49.** OREGON **50.** LETTER

51. ALGEBRA **52.** ALABAMA **53.** MISSOURI **54.** CONNECTICUT

55. **STEREO** You are going to set up a stereo system by purchasing separate components. In your price range you find 5 different receivers, 8 different compact disc players, and 12 different speaker systems. If you want one of each of these components, how many different stereo systems are possible?

56. **PIZZA** A pizza shop runs a special where you can buy a large pizza with one cheese, one vegetable, and one meat for $9.00. You have a choice of 7 cheeses, 11 vegetables, and 6 meats. Additionally, you have a choice of 3 crusts and 2 sauces. How many different variations of the pizza special are possible?

57. **COMPUTER SECURITY** To keep computer files secure, many programs require the user to enter a password. The shortest allowable passwords are typically six characters long and can contain both numbers and letters. How many six-character passwords are possible if (**a**) characters can be repeated and (**b**) characters cannot be repeated?

58. **CRITICAL THINKING** Simplify the formula for ${}_nP_r$ when $r = 0$. Explain why this result makes sense.

59. **CLASS SEATING** A particular classroom has 24 seats and 24 students. Assuming the seats are not moved, how many different seating arrangements are possible? Write your answer in scientific notation.

60. **RINGING BELLS** "Ringing the changes" is a process where the bells in a tower are rung in all possible permutations. Westminster Abbey has 10 bells in its tower. In how many ways can its bells be rung?

STUDENT HELP

INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 61.

61. **PLAY AUDITIONS** Auditions are being held for the play shown. How many ways can the roles be assigned if (**a**) 6 people audition and (**b**) 9 people audition?

62. **WINDOW DISPLAY** A music store wants to display 3 identical keyboards, 2 identical trumpets, and 2 identical guitars in its store window. How many distinguishable displays are possible?

63. **DOG SHOW** In a dog show how many ways can 3 Chihuahuas, 5 Labradors, 4 poodles, and 3 beagles line up in front of the judges if the dogs of the same breed are considered identical?

64. **CRITICAL THINKING** Find the number of permutations of n objects taken $n - 1$ at a time for any positive integer n. Compare this answer with the number of permutations of all n objects. Does this make sense? Explain.

12.2 Combinations and the Binomial Theorem

What you should learn

GOAL 1 Use combinations to count the number of ways an event can happen, as applied in **Ex. 55**.

GOAL 2 Use the binomial theorem to expand a binomial that is raised to a power.

Why you should learn it

▼ To solve **real-life** problems, such as finding the number of different combinations of plays you can attend in **Example 3**.

REAL LIFE

GOAL 1 Using Combinations

In Lesson 12.1 you learned that order is important for some counting problems. For other counting problems, order is not important. For instance, in most card games the order in which your cards are dealt is not important. After your cards are dealt, reordering them does not change your card hand. These unordered groupings are called *combinations*. A **combination** is a selection of r objects from a group of n objects where the order is not important.

COMBINATIONS OF n OBJECTS TAKEN r AT A TIME

The number of combinations of r objects taken from a group of n distinct objects is denoted by ${}_nC_r$ and is given by:

$${}_nC_r = \frac{n!}{(n-r)! \cdot r!}$$

For instance, the number of combinations of 2 objects taken from a group of 5 objects is ${}_5C_2 = \frac{5!}{3! \cdot 2!} = 10$.

EXAMPLE 1 *Finding Combinations*

A standard deck of 52 playing cards has 4 suits with 13 different cards in each suit as shown.

Standard 52-Card Deck

K ♠	K ♣	K ♦	K ♥
Q ♠	Q ♣	Q ♦	Q ♥
J ♠	J ♣	J ♦	J ♥
10 ♠	10 ♣	10 ♦	10 ♥
9 ♠	9 ♣	9 ♦	9 ♥
8 ♠	8 ♣	8 ♦	8 ♥
7 ♠	7 ♣	7 ♦	7 ♥
6 ♠	6 ♣	6 ♦	6 ♥
5 ♠	5 ♣	5 ♦	5 ♥
4 ♠	4 ♣	4 ♦	4 ♥
3 ♠	3 ♣	3 ♦	3 ♥
2 ♠	2 ♣	2 ♦	2 ♥
A ♠	A ♣	A ♦	A ♥

a. If the order in which the cards are dealt is not important, how many different 5-card hands are possible?

b. In how many of these hands are all five cards of the same suit?

SOLUTION

a. The number of ways to choose 5 cards from a deck of 52 cards is:

$${}_{52}C_5 = \frac{52!}{47! \cdot 5!}$$

$$= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot \cancel{47!}}{\cancel{47!} \cdot 5!}$$

$$= 2{,}598{,}960$$

b. For all five cards to be the same suit, you need to choose 1 of the 4 suits and then 5 of the 13 cards in the suit. So, the number of possible hands is:

$${}_4C_1 \cdot {}_{13}C_5 = \frac{4!}{3! \cdot 1!} \cdot \frac{13!}{8! \cdot 5!} = \frac{4 \cdot \cancel{3!}}{\cancel{3!} \cdot 1!} \cdot \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot \cancel{8!}}{\cancel{8!} \cdot 5!} = 5148$$

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When finding the number of ways both an event A *and* an event B can occur, you need to multiply (as you did in part (b) of Example 1). When finding the number of ways that an event A *or* an event B can occur, you add instead.

REAL LIFE *Menu Choices*

EXAMPLE 2 *Deciding to Multiply or Add*

A restaurant serves omelets that can be ordered with any of the ingredients shown.

a. Suppose you want *exactly* 2 vegetarian ingredients and 1 meat ingredient in your omelet. How many different types of omelets can you order?

b. Suppose you can afford *at most* 3 ingredients in your omelet. How many different types of omelets can you order?

Omelets $3.00
(plus $.50 for each ingredient)

Vegetarian	Meat
green pepper	ham
red pepper	bacon
onion	sausage
mushroom	steak
tomato	
cheese	

SOLUTION

a. You can choose 2 of 6 vegetarian ingredients and 1 of 4 meat ingredients. So, the number of possible omelets is:

$$_6C_2 \cdot {_4C_1} = \frac{6!}{4! \cdot 2!} \cdot \frac{4!}{3! \cdot 1!} = 15 \cdot 4 = 60$$

b. You can order an omelet with 0, 1, 2, or 3 ingredients. Because there are 10 items to choose from, the number of possible omelets is:

$$_{10}C_0 + {_{10}C_1} + {_{10}C_2} + {_{10}C_3} = 1 + 10 + 45 + 120 = 176$$

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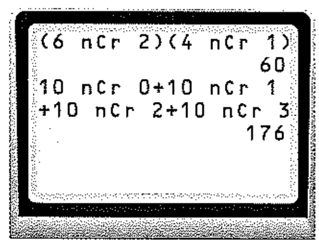
STUDENT HELP

INTERNET **KEYSTROKE HELP** Visit our Web site www.mcdougallittell.com to see keystrokes for several models of calculators.

Some calculators have special keys to evaluate combinations. The solution to Example 2 is shown.



Counting problems that involve phrases like "at least" or "at most" are sometimes easier to solve by subtracting possibilities you do not want from the total number of possibilities.

REAL LIFE *Theater*

EXAMPLE 3 *Subtracting Instead of Adding*

A theater is staging a series of 12 different plays. You want to attend *at least* 3 of the plays. How many different combinations of plays can you attend?

SOLUTION

You want to attend 3 plays, or 4 plays, or 5 plays, and so on. So, the number of combinations of plays you can attend is $_{12}C_3 + {_{12}C_4} + {_{12}C_5} + \cdots + {_{12}C_{12}}$.

Instead of adding these combinations, it is easier to use the following reasoning. For each of the 12 plays, you can choose to attend or not attend the play, so there are 2^{12} total combinations. If you attend at least 3 plays you do not attend only 0, 1, or 2 plays. So, the number of ways you can attend at least 3 plays is:

$$2^{12} - ({_{12}C_0} + {_{12}C_1} + {_{12}C_2}) = 4096 - (1 + 12 + 66) = 4017$$

Guided Practice

Vocabulary Check ✓

1. Explain the difference between a permutation and a combination.

Concept Check ✓

2. Describe a situation in which to find the total number of possibilities you would (**a**) add two combinations and (**b**) multiply two combinations.

3. Write the expansions for $(x + y)^4$ and $(x - y)^4$. How are they similar? How are they different?

4. **ERROR ANALYSIS** What error was made in the calculation of $_{10}C_6$? Explain.

$$_{10}C_6 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 7$$

Skill Check ✓

Find the number of combinations of *n* objects taken *r* at a time.

5. $n = 8, r = 2$
6. $n = 6, r = 5$
7. $n = 5, r = 1$
8. $n = 9, r = 9$

Expand the power of the binomial.

9. $(x + y)^3$
10. $(x + 1)^4$
11. $(2x + 4)^3$
12. $(2x + 3y)^5$
13. $(x - y)^5$
14. $(x - 2)^3$
15. $(3x - 1)^4$
16. $(4x - 4y)^3$

17. Complete this equation:

$$(x + 3y)^5 = x^5 + 15x^4y + 90x^3y^2 + \underline{?}\, x^2y^3 + 405xy^4 + \underline{?}\, y^5$$

Practice and Applications

STUDENT HELP

Extra Practice to help you master skills is on p. 956.

COMBINATIONS Find the number of combinations.

18. $_{10}C_2$
19. $_8C_5$
20. $_5C_2$
21. $_8C_6$
22. $_{12}C_4$
23. $_{12}C_{12}$
24. $_{14}C_6$
25. $_{11}C_3$

CARD HANDS In Exercises 26–30, find the number of possible 5-card hands that contain the cards specified.

26. 5 face cards (either kings, queens, or jacks)
27. 4 aces and 1 other card
28. 1 ace and 4 other cards (none of which are aces)
29. 2 aces and 3 kings
30. 4 of one kind (kings, queens, and so on) and 1 of a different kind

31. **PASCAL'S TRIANGLE** Copy Pascal's triangle on page 710 and add the rows for $n = 6$ and $n = 7$ to it.

PASCAL'S TRIANGLE Use the rows of Pascal's triangle from Exercise 31 to write the binomial expansion.

32. $(x + 4)^6$
33. $(x - 3y)^6$
34. $(x^2 + y)^7$
35. $(2x - y^3)^7$

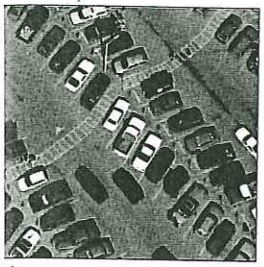
BINOMIAL THEOREM Use the binomial theorem to write the binomial expansion.

36. $(x - 2)^3$
37. $(x + 4)^5$
38. $(x + 3y)^4$
39. $(2x - y)^6$
40. $(x^3 + 3)^5$
41. $(3x^2 - 3)^4$
42. $(2x - y^2)^7$
43. $(x^3 + y^2)^3$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 18–30, 47, 48
Example 2: Exs. 49–52
Example 3: Exs. 53, 54
Examples 4–7: Exs. 31–43
Example 8: Exs. 44–46

FOCUS ON APPLICATIONS



CARS A 1998 survey showed that basic car colors, white is most popular color for -size cars with 18.8% of vote. Green came in ond with 16.4% of the .

APPLICATION LINK w.mcdougallittell.com

44. Find the coefficient of x^5 in the expansion of $(x - 3)^7$.

45. Find the coefficient of x^4 in the expansion of $(x + 2)^8$.

46. Find the coefficient of x^6 in the expansion of $(x^2 + 4)^{10}$.

47. **NOVELS** Your English teacher has asked you to select 3 novels from a list of 10 to read as an independent project. In how many ways can you choose which books to read?

48. **GAMES** Your friend is having a party and has 15 games to choose from. There is enough time to play 4 games. In how many ways can you choose which games to play?

49. **CARS** You are buying a new car. There are 7 different colors to choose from and 10 different types of optional equipment you can buy. You can choose only 1 color for your car and can afford only 2 of the options. How many combinations are there for your car?

50. **ART CONTEST** There are 6 artists each presenting 5 works of art in an art contest. The 4 works judged best will be displayed in a local gallery. In how many ways can these 4 works all be chosen from the same artist's collection?

51. **LOGICAL REASONING** Look back at Example 2. Suppose you can afford at most 7 ingredients. How many different types of omelets can you order?

52. **AMUSEMENT PARKS** An amusement park has 20 different rides. You want to ride at least 15 of them. How many different combinations of rides can you go on?

53. **FISH** From the list of different species of fish shown, an aquarium enthusiast is interested in knowing how compatible any group of 3 or more different species are. How many different combinations are there to consider?

On Sale This Month

Freshwater Tropical Fish

Neon Tetras	*Black Mollies*
Tiger Barbs	*Zebra fish*
Red Platys	*Bala Sharks*
Angelfish	*Lyretails*
Blue Gouramis	*Catfish*

54. **CONCERTS** A summer concert series has 12 different performing artists. You decide to attend at least 4 of the concerts. How many different combinations of concerts can you attend?

CRITICAL THINKING Decide whether the problem requires combinations or permutations to find the answer. Then solve the problem.

55. **MARCHING BAND** Eight members of a school marching band are auditioning for 3 drum major positions. In how many ways can students be chosen to be drum majors?

56. **YEARBOOK** Your school yearbook has an editor-in-chief and an assistant editor-in-chief. The staff of the yearbook has 15 students. In how many ways can students be chosen for these 2 positions?

57. **RELAY RACES** A relay race has 4 runners who run different parts of the race. There are 16 students on your track team. In how many ways can your coach select students to compete in the race?

58. **COLLEGE COURSES** You must take 6 elective classes to meet your graduation requirements for college. There are 12 classes that you are interested in. In how many ways can you select your elective classes?