

Section 7.1

Summary

AP STAT
CH 9 TOPICS

PRACTICE

- A **parameter** is a number that describes a population. To estimate an unknown parameter, use a **statistic** calculated from a sample.
- The **population distribution** of a variable describes the values of the variable for all individuals in a population. The **sampling distribution** of a statistic describes the values of the statistic in all possible samples of the same size from the same population. Don't confuse the sampling distribution with the **distribution of sample data**, which gives the values of the variable for individuals in a particular sample.
- A statistic can be an **unbiased estimator** or a **biased estimator** of a parameter. A statistic is an unbiased estimator if the center (mean) of its sampling distribution is equal to the true value of the parameter.
- The **variability** of a statistic is described by the spread of its sampling distribution. Larger samples give less variability.
- When trying to estimate a parameter, choose a statistic with low or no bias and minimum variability.

Section 7.1

Exercises

For Exercises 1–6, identify the population, the parameter, the sample, and the statistic in each setting.

- 1. Healthy living** From a large group of people who signed a card saying they intended to quit smoking, 1000 people were selected at random. It turned out that 210 (21%) of these individuals had not smoked over the past 6 months.
- 2. Unemployment** Each month, the Current Population Survey interviews about 60,000 randomly selected U.S. adults. One of their goals is to estimate the national unemployment rate. In October 2016, 4.9% of those interviewed were unemployed.
- 3. Fillings** How much do prices vary for filling a cavity? To find out, an insurance company randomly selects 10 dental practices in California and asks for the cash (non-insurance) price for this procedure at each practice. The interquartile range is \$74.
- 4. Warm turkey** Tom is cooking a large turkey breast for a holiday meal. He wants to be sure that the turkey is safe to eat, which requires a minimum internal temperature of 165°F. Tom uses a thermometer to measure the temperature of the turkey meat at four randomly chosen points. The minimum reading is 170°F.
- 5. Iced tea** On Tuesday, the bottles of Arizona Iced Tea filled in a plant were supposed to contain an average of 20 ounces of iced tea. Quality control inspectors selected 50 bottles at random from the day's production. These bottles contained an average of 19.6 ounces of iced tea.

- 6. Bearings** A production run of ball bearings is supposed to have a mean diameter of 2.5000 centimeters (cm). An inspector chooses 100 bearings at random from the run. These bearings have mean diameter 2.5009 cm.

Exercises 7–10 refer to the small population of 5 students in the table.

Name	Gender	Quiz score
Abigail	Female	10
Bobby	Male	5
Carlos	Male	10
DeAnna	Female	7
Emily	Female	9

- 7. Sample means** List all 10 possible SRSs of size $n = 2$, calculate the mean quiz score for each sample, and display the sampling distribution of the sample mean on a dotplot.
- 8. Sample minimums** List all 10 possible SRSs of size $n = 3$, calculate the minimum quiz score for each sample, and display the sampling distribution of the sample minimum on a dotplot.
- 9. Sample proportions** List all 10 possible SRSs of size $n = 2$, calculate the proportion of females for each sample, and display the sampling distribution of the sample proportion on a dotplot.
- 10. Sample medians** List all 10 possible SRSs of size $n = 3$, calculate the median quiz score for each sample, and display the sampling distribution of the sample median on a dotplot.

1. **Doing homework** A school newspaper article claims that 60% of the students at a large high school completed their assigned homework last week. Assume that this claim is true for the 2000 students at the school.

-) Make a bar graph of the population distribution.
-) Imagine one possible SRS of size 100 from this population. Sketch a bar graph of the distribution of sample data.

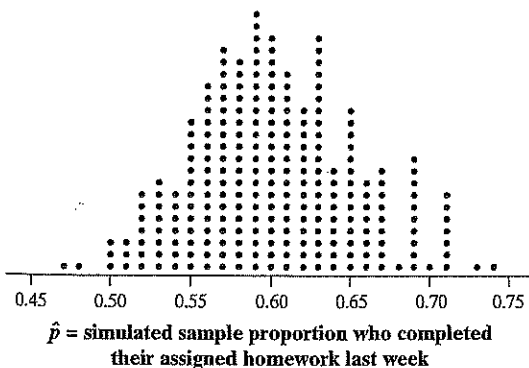
2. **Tall girls** According to the National Center for Health Statistics, the distribution of height for 16-year-old females is modeled well by a Normal density curve with mean $\mu = 64$ inches and standard deviation $\sigma = 2.5$ inches. Assume this claim is true for the three hundred 16-year-old females at a large high school.

-) Make a graph of the population distribution.
-) Imagine one possible SRS of size 20 from this population. Sketch a dotplot of the distribution of sample data.

3. **More homework** Some skeptical AP[®] Statistics students want to investigate the newspaper's claim in Exercise 1.1, so they choose an SRS of 100 students from the school to interview. In their sample, 45 students completed their homework last week. Does this provide convincing evidence that less than 60% of all students at the school completed their assigned homework last week?

-) What is the evidence that less than 60% of all students completed their assigned homework last week?
-) Provide two explanations for the evidence described in part (a).

We used technology to simulate choosing 250 SRSs of size $n = 100$ from a population of 2000 students where 60% completed their assigned homework last week. The dotplot shows \hat{p} = the sample proportion of students who completed their assigned homework last week for each of the 250 simulated samples.



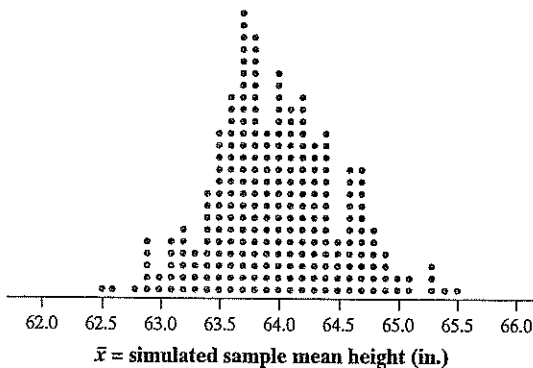
-) There is one dot on the graph at 0.73. Explain what this value represents.

- (d) Would it be surprising to get a sample proportion of $\hat{p} = 0.45$ or smaller in an SRS of size 100 when $p = 0.60$? Justify your answer.
- (e) Based on your previous answers, is there convincing evidence that less than 60% of all students at the school completed their assigned homework last week? Explain your reasoning.

14. **Tall girls?** To see if the claim made in Exercise 12 is true at their high school, an AP[®] Statistics class chooses an SRS of twenty 16-year-old females at the school and measures their heights. In their sample, the mean height is 64.7 inches. Does this provide convincing evidence that 16-year-old females at this school are taller than 64 inches, on average?

- (a) What is the evidence that the average height of all 16-year-old females at this school is greater than 64 inches, on average?
- (b) Provide two explanations for the evidence described in part (a).

We used technology to simulate choosing 250 SRSs of size $n = 20$ from a population of three hundred 16-year-old females whose heights follow a Normal distribution with mean $\mu = 64$ inches and standard deviation $\sigma = 2.5$ inches. The dotplot shows \bar{x} = the sample mean height for each of the 250 simulated samples.



- (c) There is one dot on the graph at 62.5. Explain what this value represents.
- (d) Would it be surprising to get a sample mean of $\bar{x} = 64.7$ or larger in an SRS of size 20 when $\mu = 64$ inches and $\sigma = 2.5$ inches? Justify your answer.
- (e) Based on your previous answers, is there convincing evidence that the average height of all 16-year-old females at this school is greater than 64 inches? Explain your reasoning.

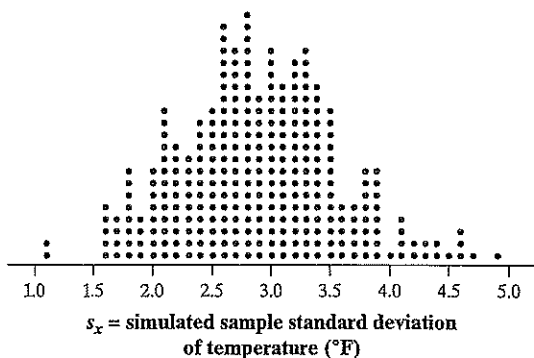
15. **Even more homework** Refer to Exercises 11 and 13. Suppose that the sample proportion of students who did all their assigned homework last week is $\hat{p} = 57/100 = 0.57$. Would this sample proportion provide convincing evidence that less than 60% of all

students at the school completed all their assigned homework last week? Explain your reasoning.

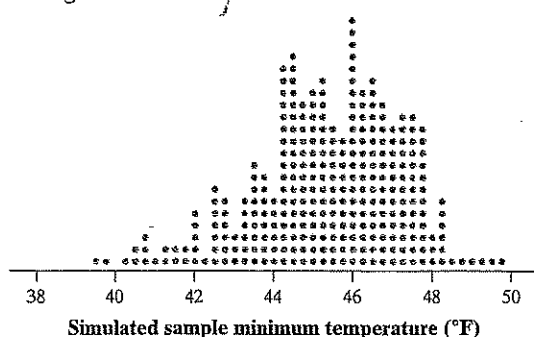
16. **Even more tall girls** Refer to Exercises 12 and 14. Suppose that the sample mean height of the twenty 16-year-old females is $\bar{x} = 65.8$ inches. Would this sample mean provide convincing evidence that the average height of all 16-year-old females at this school is greater than 64 inches? Explain your reasoning.

Exercises 17 and 18 refer to the following setting. During the winter months, outside temperatures at the Starneses' cabin in Colorado can stay well below freezing (32°F, or 0°C) for weeks at a time. To prevent the pipes from freezing, Mrs. Starnes sets the thermostat at 50°F. The manufacturer claims that the thermostat allows variation in home temperature that follows a Normal distribution with $\sigma = 3^\circ\text{F}$. To test this claim, Mrs. Starnes programs her digital thermometer to take an SRS of $n = 10$ readings during a 24-hour period. Suppose the thermostat is working properly and that the actual temperatures in the cabin vary according to a Normal distribution with mean $\mu = 50^\circ\text{F}$ and standard deviation $\sigma = 3^\circ\text{F}$.

17. **Cold cabin?** The dotplot shows the results of taking 300 SRSs of 10 temperature readings from a Normal population with $\mu = 50$ and $\sigma = 3$ and recording the sample standard deviation s_x each time. Suppose that the standard deviation from an actual sample is $s_x = 5^\circ\text{F}$. What would you conclude about the thermostat manufacturer's claim? Explain your reasoning.



18. **Really cold cabin** The dotplot shows the results of taking 300 SRSs of 10 temperature readings from a Normal population with $\mu = 50$ and $\sigma = 3$ and recording the sample minimum each time. Suppose that the minimum of an actual sample is 40°F . What would you conclude about the thermostat manufacturer's claim? Explain your reasoning.

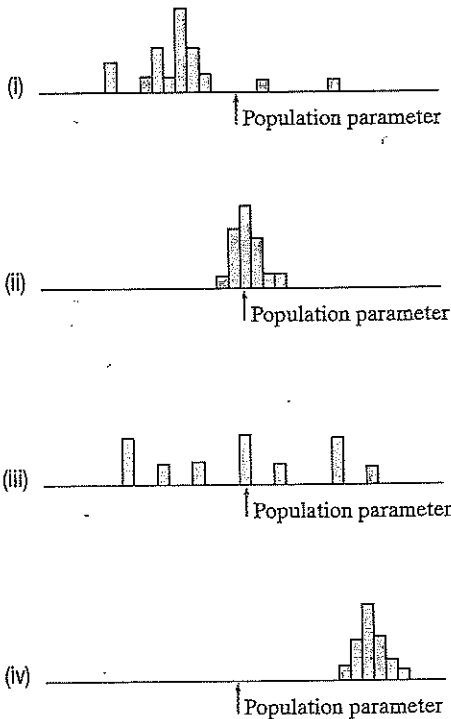


Exercises 19–22 refer to the small population of 4 cars listed in the table.

Color	Age (years)
Red	1
White	5
Silver	8
Red	20

19. **Sample proportions** List all 6 possible SRSs of size $n = 2$, calculate the proportion of red cars in the sample, and display the sampling distribution of the sample proportion on a dotplot. Is the sample proportion an unbiased estimator of the population proportion? Explain your answer.
20. **Sample minimums** List all 6 possible SRSs of size $n = 2$, calculate the minimum age for each sample, and display the sampling distribution of the sample minimum on a dotplot. Is the sample minimum an unbiased estimator of the population minimum? Explain your answer.
21. **More sample proportions** List all 4 possible SRSs of size $n = 3$, calculate the proportion of red cars in the sample, and display the sampling distribution of the sample proportion on a dotplot with the same scale as the dotplot in Exercise 19. How does the variability of this sampling distribution compare with the variability of the sampling distribution from Exercise 19? What does this indicate about increasing the sample size?
22. **More sample minimums** List all 4 possible SRSs of size $n = 3$, calculate the minimum age for each sample, and display the sampling distribution of the sample minimum on a dotplot with the same scale as the dotplot in Exercise 20. How does the variability of this sampling distribution compare with the variability of the sampling distribution from Exercise 20? What does this indicate about increasing the sample size?
23. **A sample of teens** A study of the health of teenagers plans to measure the blood cholesterol levels of an SRS of 13- to 16-year-olds. The researchers will report the mean \bar{x} from their sample as an estimate of the mean cholesterol level μ in this population. Explain to someone who knows little about statistics what it means to say that \bar{x} is an unbiased estimator of μ .
24. **Predict the election** A polling organization plans to ask a random sample of likely voters who they plan to vote for in an upcoming election. The researchers will report the sample proportion \hat{p} that favors the incumbent as an estimate of the population proportion p that favors the incumbent. Explain to someone who knows little about statistics what it means to say that \hat{p} is an unbiased estimator of p .

Bias and variability The figure shows approximate sampling distributions of 4 different statistics intended to estimate the same parameter.



Which statistics are unbiased estimators? Justify your answer.

Which statistic does the best job of estimating the parameter? Explain your answer.

Multiple Choice: Select the best answer for Exercises 26–30.

6. At a particular college, 78% of all students are receiving some kind of financial aid. The school newspaper selects a random sample of 100 students and 72% of the respondents say they are receiving some sort of financial aid. Which of the following is true?

- (a) 78% is a population and 72% is a sample.
- (b) 72% is a population and 78% is a sample.
- (c) 78% is a parameter and 72% is a statistic.
- (d) 72% is a parameter and 78% is a statistic.
- (e) 72% is a parameter and 100 is a statistic.

7. A statistic is an unbiased estimator of a parameter when

- (a) the statistic is calculated from a random sample.
- (b) in a single sample, the value of the statistic is equal to the value of the parameter.
- (c) in many samples, the values of the statistic are very close to the value of the parameter.

(d) in many samples, the values of the statistic are centered at the value of the parameter.

(e) in many samples, the distribution of the statistic has a shape that is approximately Normal.

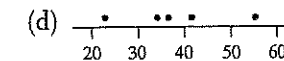
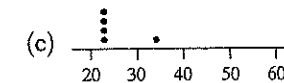
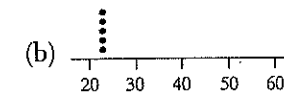
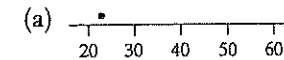
28. In a residential neighborhood, the median value of a house is \$200,000. For which of the following sample sizes is the sample median most likely to be above \$250,000?

- (a) $n = 10$
- (b) $n = 50$
- (c) $n = 100$
- (d) $n = 1000$
- (e) Impossible to determine without more information.

29. Increasing the sample size of an opinion poll will reduce the

- (a) bias of the estimates made from the data collected in the poll.
- (b) variability of the estimates made from the data collected in the poll.
- (c) effect of nonresponse on the poll.
- (d) variability of opinions in the sample.
- (e) variability of opinions in the population.

30. The math department at a small school has 5 teachers. The ages of these teachers are 23, 34, 37, 42, and 58. Suppose you select a random sample of 4 teachers and calculate the sample minimum age. Which of the following shows the sampling distribution of the sample minimum age?



(e) None of these.

Recycle and Review

31. **Dem bones (2.2)** Osteoporosis is a condition in which the bones become brittle due to loss of minerals. To diagnose osteoporosis, an elaborate apparatus measures bone mineral density (BMD). BMD is usually reported in standardized form. The standardization is based on a population of healthy young adults. The World Health Organization

$$(i) z = \frac{0.33 - 0.35}{0.0123} = -1.63 \quad \text{and} \quad z = \frac{0.37 - 0.35}{0.0123} = 1.63$$

$$\text{Using Table A: } P(0.33 \leq \hat{p} \leq 0.37) = P(-1.63 \leq z \leq 1.63) \\ = 0.9484 - 0.0516 = 0.8968$$

$$\text{Using technology: normalcdf(lower: } -1.63, \text{ upper: } 1.63, \text{ mean: } 0, \text{ SD: } 1) \\ = 0.8969$$

$$(ii) \text{ normalcdf(lower: } 0.33, \text{ upper: } 0.37, \text{ mean: } 0.35, \text{ SD: } 0.0123) = 0.8961$$

2. Perform calculations.

- (i) Standardize and use Table A or technology or
(ii) Use technology without standardizing.
Be sure to answer the question that was asked.

FOR PRACTICE TRY EXERCISE 43

In the preceding example, about 90% of all SRSs of size 1500 from this population will give a result within 2 percentage points of the truth about the population. This result also suggests that in about 90% of all SRSs of size 1500 from this population, the true proportion will be within 2 percentage points of the sample proportion. This fact will become very important in Chapter 8 when we use sample data to create an interval of plausible values for a population parameter.

Section 7.2**Summary**

- When we want information about the population proportion p of successes, we often take an SRS and use the sample proportion \hat{p} to estimate the unknown parameter p . The **sampling distribution of the sample proportion** \hat{p} describes how the statistic \hat{p} varies in all possible samples of the same size from the population.
- The **mean** of the sampling distribution of \hat{p} is $\mu_{\hat{p}} = p$. So \hat{p} is an unbiased estimator of p .
- The **standard deviation** of the sampling distribution of \hat{p} is $\sigma_{\hat{p}} = \sqrt{p(1-p)/n}$ for an SRS of size n . This formula can be used if the sample size is less than 10% of the population size (the *10% condition*).
- The sampling distribution of \hat{p} is **approximately Normal** when both $np \geq 10$ and $n(1-p) \geq 10$ (the *Large Counts condition*).

Section 7.2**Exercises**

33. **Registered voters** In a congressional district, 55% of registered voters are Democrats. A polling organization selects a random sample of 500 registered voters from this district. Let \hat{p} = the proportion of Democrats in the sample.

- Identify the mean of the sampling distribution of \hat{p} .
- Calculate and interpret the standard deviation of the sampling distribution of \hat{p} . Verify that the 10% condition is met.
- Describe the shape of the sampling distribution of \hat{p} . Justify your answer.

34. **Married with children** According to a recent U.S. Bureau of Labor Statistics report, the proportion of married couples with children in which both parents work outside the home is 59%.⁶ You select an SRS of 50 married couples with children and let \hat{p} = the sample proportion of couples in which both parents work outside the home.

- Identify the mean of the sampling distribution of \hat{p} .
- Calculate and interpret the standard deviation of the sampling distribution of \hat{p} . Verify that the 10% condition is met.
- Describe the shape of the sampling distribution of \hat{p} . Justify your answer.

35. **Orange Skittles®** The makers of Skittles claim that 20% of Skittles candies are orange. Suppose this claim is true. You select a random sample of 30 Skittles from a large bag. Let \hat{p} = the proportion of orange Skittles in the sample.

- Identify the mean of the sampling distribution of \hat{p} .
- Calculate and interpret the standard deviation of the sampling distribution of \hat{p} . Verify that the 10% condition is met.
- Describe the shape of the sampling distribution of \hat{p} . Justify your answer.

36. **Male workers** A factory employs 3000 unionized workers, 90% of whom are male. A random sample of 15 workers is selected for a survey about worker satisfaction. Let \hat{p} = the proportion of males in the sample.

- Identify the mean of the sampling distribution of \hat{p} .
- Calculate and interpret the standard deviation of the sampling distribution of \hat{p} . Verify that the 10% condition is met.
- Describe the shape of the sampling distribution of \hat{p} . Justify your answer.

37. **More Skittles®** What sample size would be required to reduce the standard deviation of the sampling distribution to one-half the value you found in Exercise 35(b)? Justify your answer.

38. **More workers** What sample size would be required to reduce the standard deviation of the sampling distribution to one-third the value you found in Exercise 36(b)? Justify your answer.

39. **Airport security** The Transportation Security Administration (TSA) is responsible for airport safety. On some flights, TSA officers randomly select passengers for an extra security check before boarding. One such flight had 76 passengers—12 in first class and 64 in coach class. TSA officers selected an SRS of 10 passengers for screening. Let \hat{p} be the proportion of first-class passengers in the sample.

- Is the 10% condition met in this case? Justify your answer.
- Is the Large Counts condition met in this case? Justify your answer.

40. **Scrabble®** In the game of Scrabble, each player begins by drawing 7 tiles from a bag containing 100 tiles. There are 42 vowels, 56 consonants, and 2 blank tiles in the bag. Cait chooses an SRS of 7 tiles. Let \hat{p} be the proportion of vowels in her sample.

- Is the 10% condition met in this case? Justify your answer.
- Is the Large Counts condition met in this case? Justify your answer.

41. **Do you drink the cereal milk?** A *USA Today* poll asked a random sample of 1012 U.S. adults what they do with the milk in the bowl after they have eaten the cereal. Let \hat{p} be the proportion of people in the sample who drink the cereal milk. A spokesman for the dairy industry claims that 70% of all U.S. adults drink the cereal milk. Suppose this claim is true.

- What is the mean of the sampling distribution of \hat{p} ?
- Find the standard deviation of the sampling distribution of \hat{p} . Verify that the 10% condition is met.

(c) Verify that the sampling distribution of \hat{p} is approximately Normal.

(d) Of the poll respondents, 67% said that they drink the cereal milk. Find the probability of obtaining a sample of 1012 adults in which 67% or fewer say they drink the cereal milk, assuming the milk industry spokesman's claim is true.

(e) Does this poll give convincing evidence against the spokesman's claim? Explain your reasoning.

42. **Do you go to church?** The Gallup Poll asked a random sample of 1785 adults if they attended church during the past week. Let \hat{p} be the proportion of people in the sample who attended church. A newspaper report claims that 40% of all U.S. adults went to church last week. Suppose this claim is true.

- What is the mean of the sampling distribution of \hat{p} ?
- Find the standard deviation of the sampling distribution of \hat{p} . Verify that the 10% condition is met.

(c) Verify that the sampling distribution of \hat{p} is approximately Normal.

(d) Of the poll respondents, 44% said they did attend church last week. Find the probability of obtaining a sample of 1785 adults in which 44% or more say they attended church last week, assuming the newspaper report's claim is true.

(e) Does this poll give convincing evidence against the newspaper's claim? Explain your reasoning.

43. **Students on diets** Suppose that 70% of college women have been on a diet within the past 12 months.

▶ A sample survey interviews an SRS of 267 college women. What is the probability that 75% or more of the women in the sample have been on a diet?

44. **Who owns a Harley?** Harley-Davidson motorcycles make up 14% of all the motorcycles registered in the United States. You plan to interview an SRS of 500 motorcycle owners. How likely is your sample to contain 20% or more who own Harleys?

45. **On-time shipping** A mail-order company advertises that it ships 90% of its orders within three working days.

- Choose an SRS of size n from a population with mean μ and standard deviation σ . If the population distribution is Normal, then so is the sampling distribution of the sample mean \bar{x} . If the population distribution is not Normal, the central limit theorem (CLT) states that when n is large, the sampling distribution of \bar{x} is approximately Normal.
- In some cases, we can use a Normal distribution to calculate probabilities for events involving \bar{x} .
 - If the population distribution is Normal, so is the sampling distribution of \bar{x} .
 - If $n \geq 30$, the sampling distribution of \bar{x} will be approximately Normal in most cases.

Section 7.3 Exercises

53. **Songs on an iPod** David's iPod has about 10,000 songs. The distribution of the play times for these songs is heavily skewed to the right with a mean of 225 seconds and a standard deviation of 60 seconds. Suppose we choose an SRS of 10 songs from this population and calculate the mean play time \bar{x} of these songs.

- Identify the mean of the sampling distribution of \bar{x} .
- Calculate and interpret the standard deviation of the sampling distribution of \bar{x} . Verify that the 10% condition is met.

54. **Making auto parts** A grinding machine in an auto parts plant prepares axles with a target diameter $\mu = 40.125$ millimeters (mm). The machine has some variability, so the standard deviation of the diameters is $\sigma = 0.002$ mm. The machine operator inspects a random sample of 4 axles each hour for quality control purposes and records the sample mean diameter \bar{x} . Assume the machine is working properly.

- Identify the mean of the sampling distribution of \bar{x} .
- Calculate and interpret the standard deviation of the sampling distribution of \bar{x} . Verify that the 10% condition is met.

55. **Songs on an iPod** Refer to Exercise 53. How many songs would you need to sample if you wanted the standard deviation of the sampling distribution of \bar{x} to be 10 seconds? Justify your answer.

56. **Making auto parts** Refer to Exercise 54. How many axles would you need to sample if you wanted the standard deviation of the sampling distribution of \bar{x} to be 0.0005 mm? Justify your answer.

57. **Bottling cola** A bottling company uses a filling machine to fill plastic bottles with cola. The bottles are supposed to contain 300 milliliters (ml). In fact, the contents vary

according to a Normal distribution with mean $\mu = 298$ ml and standard deviation $\sigma = 3$ ml.

- What is the probability that a randomly selected bottle contains less than 295 ml?
- What is the probability that the mean contents of six randomly selected bottles is less than 295 ml?

58. **Cereal** A company's cereal boxes advertise that each box contains 9.65 ounces of cereal. In fact, the amount of cereal in a randomly selected box follows a Normal distribution with mean $\mu = 9.70$ ounces and standard deviation $\sigma = 0.03$ ounce.

- What is the probability that a randomly selected box of the cereal contains less than 9.65 ounces of cereal?
- Now take an SRS of 5 boxes. What is the probability that the mean amount of cereal in these boxes is less than 9.65 ounces?

59. **Cholesterol** Suppose that the blood cholesterol level of all men aged 20 to 34 follows the Normal distribution with mean $\mu = 188$ milligrams per deciliter (mg/dl) and standard deviation $\sigma = 41$ mg/dl.

- Choose an SRS of 100 men from this population. Describe the sampling distribution of \bar{x} .
- Find the probability that \bar{x} estimates μ within ± 3 mg/dl. (This is the probability that \bar{x} takes a value between 185 and 191 mg/dl.)
- Choose an SRS of 1000 men from this population. Now what is the probability that \bar{x} falls within ± 3 mg/dl of μ ? In what sense is the larger sample "better"?

60. **Finch beaks** One dimension of bird beaks is "depth"—the height of the beak where it arises from the bird's head. During a research study on one island in the Galapagos archipelago, the beak depth of all

Medium Ground Finches on the island was found to be Normally distributed with mean $\mu = 9.5$ millimeters (mm) and standard deviation $\sigma = 1.0$ mm.⁹

- Choose an SRS of 5 Medium Ground Finches from this population. Describe the sampling distribution of \bar{x} .
- Find the probability that \bar{x} estimates μ within ± 0.5 mm. (This is the probability that \bar{x} takes a value between 9 and 10 mm.)
- Choose an SRS of 50 Medium Ground Finches from this population. Now what is the probability that \bar{x} falls within ± 0.5 mm of μ ? In what sense is the larger sample "better"?

61. **Dead battery?** A car company claims that the lifetime of its batteries varies from car to car according to a Normal distribution with mean $\mu = 48$ months and standard deviation $\sigma = 8.2$ months. A consumer organization installs this type of battery in an SRS of 8 cars and calculates $\bar{x} = 42.2$ months.

- Find the probability that the sample mean lifetime is 42.2 months or less if the company's claim is true.
- Based on your answer to part (a), is there convincing evidence that the company is overstating the average lifetime of its batteries?

62. **Foiled again?** The manufacturer of a certain brand of aluminum foil claims that the amount of foil on each roll follows a Normal distribution with a mean of 250 square feet (ft^2) and a standard deviation of 2 ft^2 . To test this claim, a restaurant randomly selects 10 rolls of this aluminum foil and carefully measures the mean area to be $\bar{x} = 249.6$ ft^2 .

- Find the probability that the sample mean area is 249.6 ft^2 or less if the manufacturer's claim is true.
- Based on your answer to part (a), is there convincing evidence that the company is overstating the average area of its aluminum foil rolls?

63. **Songs on an iPod** David's iPod has about 10,000 songs. The distribution of the play times for these songs is heavily skewed to the right with a mean of 225 seconds and a standard deviation of 60 seconds.

- Describe the shape of the sampling distribution of \bar{x} for SRSs of size $n = 5$ from the population of songs on David's iPod. Justify your answer.
- Describe the shape of the sampling distribution of \bar{x} for SRSs of size $n = 100$ from the population of songs on David's iPod. Justify your answer.

64. **High school GPAs** The distribution of grade point average for students at a large high school is skewed to the left with a mean of 3.53 and a standard deviation of 1.02.

- Describe the shape of the sampling distribution of \bar{x} for SRSs of size $n = 4$ from the population of students at this high school. Justify your answer.

- Describe the shape of the sampling distribution of \bar{x} for SRSs of size $n = 50$ from the population of students at this high school. Justify your answer.

65. **More on insurance** An insurance company claims that in the entire population of homeowners, the mean annual loss from fire is $\mu = \$250$ and the standard deviation of the loss is $\sigma = \$5000$. The distribution of losses is strongly right-skewed: many policies have \$0 loss, but a few have large losses. The company hopes to sell 1000 of these policies for \$300 each.

- Assuming that the company's claim is true, what is the probability that the mean loss from fire is greater than \$300 for an SRS of 1000 homeowners?
- If the company wants to be 90% certain that the mean loss from fire in an SRS of 1000 homeowners is less than the amount it charges for the policy, how much should the company charge?

66. **Cash grab** At a traveling carnival, a popular game is called the "Cash Grab." In this game, participants step into a sealed booth, a powerful fan turns on, and dollar bills are dropped from the ceiling. A customer has 30 seconds to grab as much cash as possible while the dollar bills swirl around. Over time, the operators of the game have determined that the mean amount grabbed is \$13 with a standard deviation of \$9. They charge \$15 to play the game and expect to have 40 customers at their next carnival.

- What is the probability that an SRS of 40 customers grab an average of \$15 or more?
- How much should the operators charge if they want to be 95% certain that the mean amount grabbed by an SRS of 40 customers is less than what they charge to play the game?

67. **Bad carpet** The number of flaws per square yard in a type of carpet material varies with mean 1.6 flaws per square yard and standard deviation 1.2 flaws per square yard.

- Without doing any calculations, explain which event is more likely:

- randomly selecting a 1 square yard of material and finding 2 or more flaws
- randomly selecting 50 square yards of material and finding an average of 2 or more flaws

- Explain why you cannot use a Normal distribution to calculate the probability of the first event in part (a).
- Calculate the probability of the second event in part (a).

68. **How many people in a car?** A study of rush-hour traffic in San Francisco counts the number of people in each car entering a freeway at a suburban interchange. Suppose that this count has mean 1.6 and standard deviation 0.75 in the population of all cars that enter at this interchange during rush hour.

- a) Without doing any calculations, explain which event is more likely:
- randomly selecting 1 car entering this interchange during rush hour and finding 2 or more people in the car
 - randomly selecting 35 cars entering this interchange during rush hour and finding an average of 2 or more people in the cars
- b) Explain why you cannot use a Normal distribution to calculate the probability of the first event in part (a).
- c) Calculate the probability of the second event in part (a).
59. What does the CLT say? Asked what the central limit theorem says, a student replies, "As you take larger and larger samples from a population, the histogram of the sample values looks more and more Normal." Is the student right? Explain your answer.
70. What does the CLT say? Asked what the central limit theorem says, a student replies, "As you take larger and larger samples from a population, the variability of the sampling distribution of the sample mean decreases." Is the student right? Explain your answer.
71. **Airline passengers get heavier** In response to the increasing weight of airline passengers, the Federal Aviation Administration (FAA) told airlines to assume that passengers average 190 pounds in the summer, including clothes and carry-on baggage. But passengers vary, and the FAA did not specify a standard deviation. A reasonable standard deviation is 35 pounds. A commuter plane carries 30 passengers. Find the probability that the total weight of 30 randomly selected passengers exceeds 6000 pounds. (*Hint:* To calculate this probability, restate the problem in terms of the mean weight.)
72. **Lightning strikes** The number of lightning strikes on a square kilometer of open ground in a year has mean 6 and standard deviation 2.4. The National Lightning Detection Network (NLDN) uses automatic sensors to watch for lightning in 1-square-kilometer plots of land. Find the probability that the total number of lightning strikes in a random sample of 50 square-kilometer plots of land is less than 250. (*Hint:* To calculate this probability, restate the problem in terms of the mean number of strikes.)
- Multiple Choice:** Select the best answer for Exercises 73–76.
73. The distribution of scores on the mathematics part of the SAT exam in a recent year was approximately Normal with mean 515 and standard deviation 114. Imagine choosing many SRSs of 100 students who took the exam and averaging their SAT Math scores. Which of the following are the mean and standard deviation of the sampling distribution of \bar{x} ?
- (d) Mean = 515/100, SD = $114/\sqrt{100}$
- (e) Cannot be determined without knowing the 100 scores.
74. Why is it important to check the 10% condition before calculating probabilities involving \bar{x} ?
- (a) To reduce the variability of the sampling distribution of \bar{x}
- (b) To ensure that the distribution of \bar{x} is approximately Normal
- (c) To ensure that we can generalize the results to a larger population
- (d) To ensure that \bar{x} will be an unbiased estimator of μ
- (e) To ensure that the observations in the sample are close to independent
75. A machine is designed to fill 16-ounce bottles of shampoo. When the machine is working properly, the amount poured into the bottles follows a Normal distribution with mean 16.05 ounces and standard deviation 0.1 ounce. Assume that the machine is working properly. If 4 bottles are randomly selected and the number of ounces in each bottle is measured, then there is about a 95% probability that the sample mean will fall in which of the following intervals?
- (a) 16.05 to 16.15 ounces (d) 15.90 to 16.20 ounces
- (b) 16.00 to 16.10 ounces (e) 15.85 to 16.25 ounces
- (c) 15.95 to 16.15 ounces
76. The number of hours a lightbulb burns before failing varies from bulb to bulb. The population distribution of burnout times is strongly skewed to the right. The central limit theorem says that
- (a) as we look at more and more bulbs, their average burnout time gets ever closer to the mean μ for all bulbs of this type.
- (b) the average burnout time of a large number of bulbs has a sampling distribution with the same shape (strongly skewed) as the population distribution.
- (c) the average burnout time of a large number of bulbs has a sampling distribution with a similar shape but not as extreme (skewed, but not as strongly) as the population distribution.
- (d) the average burnout time of a large number of bulbs has a sampling distribution that is close to Normal.
- (e) the average burnout time of a large number of bulbs has a sampling distribution that is exactly Normal.

Recycle and Review

Exercises 77 and 78 refer to the following setting. In the language of government statistics, you are "in the labor force" if you are available for work and either working or actively seeking work. The unemployment rate is the proportion of the labor force

You select an SRS of 100 of the 5000 orders received in the past week for an audit. The audit reveals that 86 of these orders were shipped on time.

- (a) If the company really ships 90% of its orders on time, what is the probability that the proportion in an SRS of 100 orders is 0.86 or less?
- (b) Based on your answer to part (a), is there convincing evidence that less than 90% of all orders from this company are shipped within three working days? Explain your reasoning.

46. **Wait times** A hospital claims that 75% of people who come to its emergency room are seen by a doctor within 30 minutes of checking in. To verify this claim, an auditor inspects the medical records of 55 randomly selected patients who checked into the emergency room during the last year. Only 32 (58.2%) of these patients were seen by a doctor within 30 minutes of checking in.

- (a) If the wait time is less than 30 minutes for 75% of all patients in the emergency room, what is the probability that the proportion of patients who wait less than 30 minutes is 0.582 or less in a random sample of 55 patients?
- (b) Based on your answer to part (a), is there convincing evidence that less than 75% of all patients in the emergency room wait less than 30 minutes? Explain your reasoning.

Multiple Choice Select the best answer for Exercises 47–50.

Exercises 47–49 refer to the following setting. The magazine *Sports Illustrated* asked a random sample of 750 Division I college athletes, “Do you believe performance-enhancing drugs are a problem in college sports?” Suppose that 30% of all Division I athletes think that these drugs are a problem. Let \hat{p} be the sample proportion who say that these drugs are a problem.

47. Which of the following are the mean and standard deviation of the sampling distribution of the sample proportion \hat{p} ?

- (a) Mean = 0.30, SD = 0.017
 (b) Mean = 0.30, SD = 0.55
 (c) Mean = 0.30, SD = 0.0003
 (d) Mean = 225, SD = 12.5
 (e) Mean = 225, SD = 157.5

48. Decreasing the sample size from 750 to 375 would multiply the standard deviation by

- (a) 2. (d) $1/\sqrt{2}$.
 (b) $\sqrt{2}$. (e) none of these.
 (c) $1/2$.

49. The sampling distribution of \hat{p} is approximately Normal because

- (a) there are at least 7500 Division I college athletes.
 (b) $np = 225$ and $n(1 - p) = 525$ are both at least 10.
 (c) a random sample was chosen.
 (d) the athletes' responses are quantitative.
 (e) the sampling distribution of \hat{p} always has this shape.

50. In a congressional district, 55% of the registered voters are Democrats. Which of the following is equivalent to the probability of getting less than 50% Democrats in a random sample of size 100?

- (a) $P\left(z < \frac{0.50 - 0.55}{100}\right)$
 (b) $P\left(z < \frac{0.50 - 0.55}{\sqrt{\frac{0.55(0.45)}{100}}}\right)$
 (c) $P\left(z < \frac{0.55 - 0.50}{\sqrt{\frac{0.55(0.45)}{100}}}\right)$
 (d) $P\left(z < \frac{0.50 - 0.55}{\sqrt{100(0.55)(0.45)}}\right)$
 (e) $P\left(z < \frac{0.55 - 0.50}{\sqrt{100(0.55)(0.45)}}\right)$

Recycle and Review

51. **Sharing music online** (5.2, 5.3) A sample survey reports that 29% of Internet users download music files online, 21% share music files from their computers, and 12% both download and share music.⁷

- (a) Make a two-way table that displays this information.
 (b) What percent of Internet users neither download nor share music files?
 (c) Given that an Internet user downloads music files online, what is the probability that this person also shares music files?

52. **Whole grains** (4.2) A series of observational studies revealed that people who typically consume 3 servings of whole grain per day have about a 20% lower risk of dying from heart disease and about a 15% lower risk of dying from stroke or cancer than those who consume no whole grains.⁸

- (a) Explain how confounding makes it difficult to establish a cause-and-effect relationship between whole grain consumption and risk of dying from heart disease, stroke, or cancer, based on these studies.
 (b) Explain how researchers could establish a cause-and-effect relationship in this context.