

AP STAT CH 10 PRACTICE

FRAPPY! FREE RESPONSE AP® PROBLEM, YAY!

The following problem is modeled after actual AP® Statistics exam free response questions. Your task is to generate a complete, concise response in 15 minutes.

Directions: Show all your work. Indicate clearly the methods you use, because you will be scored on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

Members at a popular fitness club currently pay a \$40 per month membership fee. The owner of the club wants to raise the fee to \$50 but is concerned that some members will leave the gym if the fee increases. To investigate, the owner plans to survey a random sample of the club members and construct a 95% confidence interval for the proportion of all members who would quit if the fee was raised to \$50.

- (c) According to the club's accountant, the fee increase will be worthwhile if fewer than 20% of the members quit. According to the interval from part (b), can the owner be confident that the fee increase will be worthwhile? Explain.
- (d) One of the conditions for calculating the confidence interval in part (b) is that $n\hat{p} \geq 10$ and $n(1 - \hat{p}) \geq 10$. Explain why it is necessary to check this condition.

- (a) Explain the meaning of "95% confidence" in the context of the study.
- (b) After the owner conducted the survey, he calculated the confidence interval to be 0.18 ± 0.075 . Interpret this interval in the context of the study.

TEACHER

(a) 95% confidence indicates the level of certainty in which the pop proportion would be contained in a given interval.

(b) With 95% confidence, the true proportion of members that would quit would be between .105 and .255 (if fee raised)

(c) The owner can't be confident since the interval goes as high as 25.5% of members that would quit. 20% is about the middle so it's close to a 50% chance they would potentially lose money

(d) It is necessary to check the proportion to verify if your sample size is sufficient enough to ^{make} an inference. Based on normality

batteries last between 430 and 470 minutes." Comment on this interpretation.

- (c) Your friend, who has just started studying statistics, claims that there is a 95% probability that the mean lifetime will fall between 430 and 470 minutes. Do you agree? Explain your reasoning.
- (d) Give a statistically correct interpretation of the confidence level that could be published in a newspaper report.

R8.3 We love football! A Gallup poll conducted telephone interviews with a random sample of adults aged 18 and older. Data were obtained for 1000 people. Of these, 370 said that football is their favorite sport to watch on television.

- (a) Define the parameter p in this setting. *all adults 18 or older that would say football is their fav. sp.*
- (b) What point estimator will you use to estimate p ? What is the value of the point estimate? *$\hat{p} = .37$*
- (c) Do you believe that the value of the point estimate is equal to the value of p ? Explain your answer. *No - sampling variability*

R8.4 Running red lights A random digit dialing telephone survey of 880 drivers asked, "Recalling the last ten traffic lights you drove through, how many of them were red when you entered the intersections?" Of the 880 respondents, 171 admitted that at least one light had been red.³⁷ *check $n\hat{p} \geq 10$ $n\hat{q} \geq 10$*

- (a) Construct and interpret a 95% confidence interval for the population proportion.
- (b) Nonresponse is a practical problem for this survey—only 21.6% of calls that reached a live person were completed. Another practical problem is that people may not give truthful answers. What is the likely direction of the bias: Do you think more or fewer than 171 of the 880 respondents really ran a red light? Why? Are these sources of bias included in the margin of error?

R8.5 Engine parts A random sample of 16 of the more than 200 auto engine crankshafts produced in one day was selected. Here are measurements (in millimeters) of a critical component on these crankshafts:

224.120	224.001	224.017	223.982	223.989	223.961	223.960	224.089
223.987	223.976	223.902	223.980	224.098	224.057	223.913	223.999

8.4 a) 95% $\Rightarrow z_c = 1.96$ $\hat{p} = \frac{171}{880} = .19$ $\hat{q} = .81$

$$E = 1.96 \sqrt{\frac{(.19)(.81)}{880}} = .026 \quad (.164, .216)$$

w/ 95% confidence the true pop prop. of drivers who have run at least 1 red light is between 16.4% - 21.6% in the last 10 intersections

(b) more some will deny margin of error does not account for bias.

- (a) Construct and interpret a 95% confidence interval for the mean length of this component on all the crankshafts produced on that day.
- (b) The mean length is supposed to be $\mu = 224$ mm but can drift away from this target during production. Does your interval from part (a) suggest that the mean has drifted from 224 mm? Explain your answer.

R8.6 Do you go to church? The Gallup Poll plans to ask a random sample of adults whether they attended a religious service in the last 7 days. How large a sample would be required to obtain a margin of error of at most 0.01 in a 99% confidence interval for the population proportion who would say that they attended a religious service? *$n \geq 16590$*

R8.7 Good wood? A lab supply company sells pieces of Douglas fir 4 inches long and 1.5 inches square for force experiments in science classes. From experience, the strength of these pieces of wood follows a distribution with standard deviation 3000 pounds. You want to estimate the mean load needed to pull apart these pieces of wood to within 1000 pounds with 95% confidence. How large a sample is needed? *$n = 35$*

R8.8 It's about ME Explain how each of the following would affect the margin of error of a confidence interval, if all other things remained the same.

- (a) Increasing the confidence level *ME goes up*
- (b) Quadrupling the sample size *ME goes down*

R8.9 t time When constructing confidence intervals for a population mean, we almost always use critical values from a t distribution rather than the standard Normal distribution. *$n < 30$ skew outliers*

- (a) When is it necessary to use a t critical value rather than a z critical value when constructing a confidence interval for a population mean?
- (b) For a particular level of confidence, explain what happens to the t critical values as the degrees of freedom increase. *get smaller \rightarrow approach z*

8.5) Check conditions

t test $n = 16$ $df = 15$

95% $t^ = 2.131$ $\bar{x} = 224.002$ $s_x = .0618$*

$$E = 2.131 \left(\frac{.0618}{\sqrt{16}} \right) = .0329$$

(223.969, 224.035)

With 95% confidence the length of all crankshafts produced will be between 223.969 and 224.035 mm.

(b) Because 224 is in the interval it's plausible that it is the true mean & no real evidence of drift.

Chapter 8 AP® Statistics Practice Test

Section I: Multiple Choice *Select the best answer for each question.*

T8.1 The Gallup Poll interviews 1600 people. Of these, 18% say that they jog regularly. The news report adds: "The poll had a margin of error of plus or minus three percentage points at a 95% confidence level." You can safely conclude that

- (a) 95% of all Gallup Poll samples like this one give answers within $\pm 3\%$ of the true population value.
- (b) the percent of the population who jog is certain to be between 15% and 21%.
- (c) 95% of the population jog between 15% and 21% of the time.
- (d) we can be 95% confident that the sample proportion is captured by the confidence interval.
- (e) if Gallup took many samples, 95% of them would find that 18% of the people in the sample jog.

T8.2 The weights (in pounds) of three adult males are 160, 215, and 195. What is the standard error of the mean for these data?

- (a) 90
- (b) 27.84
- (c) 22.73
- (d) 16.07
- (e) 13.13

T8.3 In preparing to construct a one-sample t interval for a population mean, suppose we are not sure if the population distribution is Normal. In which of the following circumstances would we *not* be safe constructing the interval based on an SRS of size 14 from the population?

- (a) A stemplot of the data is roughly bell-shaped.
- (b) A histogram of the data shows slight skewness.
- (c) A boxplot shows that the values above the median are much more variable than the values below the median.
- (d) The sample standard deviation is large.
- (e) The sample standard deviation is small.

T8.4 Many television viewers express doubts about the validity of certain commercials. In an attempt to answer their critics, Timex Group USA wishes to estimate the true proportion p of all consumers who believe what is shown in Timex television commercials. What is the *smallest* number of consumers that Timex can survey to guarantee a margin of error of 0.05 or less at a 99% confidence level?

- (a) 550
- (b) 600
- (c) 650
- (d) 700
- (e) 750

T8.5 You want to compute a 90% confidence interval for the mean of a population with an unknown population standard deviation. The sample size is 30. What critical value should you use for this interval?

- (a) 1.645
- (b) 1.699
- (c) 1.697
- (d) 1.96
- (e) 2.045

T8.6 A radio talk show host with a large audience is interested in the proportion p of adults in his listening area who think the drinking age should be lowered to eighteen. To find this out, he poses the following question to his listeners: "Do you think that the drinking age should be reduced to eighteen in light of the fact that 18-year-olds are eligible for military service?" He asks listeners to go to his website and vote "Yes" if they agree the drinking age should be lowered and "No" if not. Of the 100 people who voted, 70 answered "Yes." Which of the following conditions are violated?

I. Random II. 10% III. Large Counts

- (a) I only
- (b) II only
- (c) III only
- (d) I and II only
- (e) I, II, and III

T8.7 A 90% confidence interval for the mean μ of a population is computed from a random sample and is found to be 90 ± 30 . Which of the following *could* be the 95% confidence interval based on the same data?

- (a) 90 ± 21
- (b) 90 ± 30
- (c) 90 ± 39
- (d) 90 ± 70
- (e) Without knowing the sample size, any of the above answers could be the 95% confidence interval.

T8.8 Suppose we want a 90% confidence interval for the average amount spent on books by freshmen in their first year at a major university. The interval is to have a margin of error of at most \$2. Based on last year's book sales, we estimate that the standard deviation of the amount spent will be close to \$30. The number of observations required is closest to which of the following?

- (a) 25
- (b) 30
- (c) 225
- (d) 609
- (e) 865

$$n = \left(\frac{z^*}{E} \right)^2 \cdot \frac{pq}{n} \quad E = t \quad \sqrt{\frac{pq}{n}} \quad \left(\frac{E}{t} \right)^2 = \frac{pq}{n} \cdot \frac{n-1}{E^2}$$

$$\left(\frac{(1.64)(30)}{2} \right)^2$$

T8.9 A telephone poll of an SRS of 1234 adults found that 62% are generally satisfied with their lives. The announced margin of error for the poll was 3%. Does the margin of error account for the fact that some adults do not have telephones?

- (a) Yes; the margin of error accounts for all sources of error in the poll.
- (b) Yes; taking an SRS eliminates any possible bias in estimating the population proportion.
- (c) Yes; the margin of error accounts for undercoverage but not nonresponse.
- (d) No; the margin of error accounts for nonresponse but not undercoverage.
- (e)** No; the margin of error only accounts for sampling variability.

T8.10 A Census Bureau report on the income of Americans says that with 90% confidence the median income of all U.S. households in a recent year was \$57,005, with a margin of error of \$742. Which of the following is the most appropriate conclusion?

- (a) 90% of all households had incomes in the interval $\$57,005 \pm \742 .
- (b) We can be sure that the median income for all households in the country lies in the interval $\$57,005 \pm \742 .
- (c) 90% of the households in the sample interviewed by the Census Bureau had incomes in the interval $\$57,005 \pm \742 .
- (d)** The Census Bureau got the result $\$57,005 \pm \742 using a method that will capture the true median income 90% of the time when used repeatedly.
- (e) 90% of all possible samples of this same size would result in a sample median that falls within \$742 of \$57,005.

Section II: Free Response Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations.

T8.11 The U.S. Forest Service is considering additional restrictions on the number of vehicles allowed to enter Yellowstone National Park. To assess public reaction, the service asks a random sample of 150 visitors if they favor the proposal. Of these, 89 say "Yes."

- (a) Construct and interpret a 99% confidence interval for the proportion of all visitors to Yellowstone who favor the restrictions. **(.490, .696)**
- (b) Based on your work in part (a), is there convincing evidence that more than half of all visitors to Yellowstone National Park favor the proposal? Justify your answer. **B/c values smaller than .5 in interval can not conclude**

T8.12 Many people have asked the question, but few have been patient enough to collect the data. How many licks does it take to get to the center of a Tootsie Pop? After some intense research, a researcher revealed

a 95% confidence interval for the mean number of licks to be 317.64 licks to 394.56 licks.³⁸

- (a) Interpret the confidence interval.
- (b) Calculate the point estimate and margin of error used to construct the interval. **356.1 / 38.46**
- (c) Name two things the researcher could do to decrease the margin of error. Discuss a drawback of each. **sample size / decrease conf. level**

T8.13 A milk processor monitors the number of bacteria per milliliter in raw milk received at the factory. A random sample of 10 one-milliliter specimens of milk supplied by one producer gives the following data:

5370 4890 5100 4500 5260 5150 4900 4760 4700 4870

Construct and interpret a 90% confidence interval for the population mean μ .

$n=10$ less than 10% of all specimens
 $\bar{x}=4950$ $s_x=268.5$ $n=10$ $df=9$
 $t^* = 1.833$ **(4794.37, 5105.63)**