Chapter 10: Estimating with Confidence

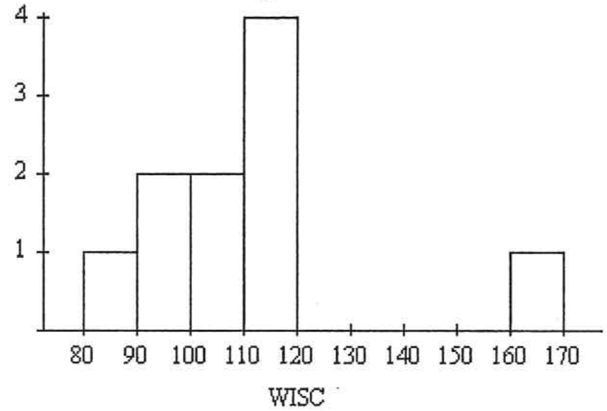
1. A 95% confidence interval for the mean μ of a population is computed from a random sample and found to be 9 ± 3. We may conclude that
   A) there is a 95% probability that μ is between 6 and 12.
   B) 95% of values sampled are between 6 and 12.
   C) if we took many, many additional random samples and from each computed a 95% confidence interval for μ, approximately 95% of these intervals would contain μ.
   D) there is a 95% probability that the true mean is 9 and a 95% chance that the true margin of error is 3.
   E) all of the above are true.

2. The records of all 100 postal employees at a postal station in a large city showed that the average amount of time these employees had worked for the U.S. Postal Service was $\overline{X}$ = 8 years. Assume that we know that the standard deviation of the amount of time U.S. Postal Service employees have spent with the Postal Service is approximately normal with standard deviation σ = 5 years. Based on these data, a 95% confidence interval for the mean number of years μ that a U.S. Postal Service employee has spent with the Postal Service would be
   A) 8 ± 0.82. B) 8 ± 0.98. C) 8 ± 1.96. D) 8 ± 9.80. E) 8 ± 0.098.

$1.96\left(\frac{5}{\sqrt{100}}\right)$

.98

3. The scores of a certain population on the Wechsler Intelligence Scale for Children (WISC) are thought to be normally distributed with mean μ and standard deviation $\sigma =$ 10. A simple random sample of 10 children from this population is taken, and each is given the WISC. The 95% confidence interval for μ, $\bar{X} \pm 1.96\frac{10}{\sqrt{10}}$, is computed from these scores. A histogram of the 10 WISC scores is given below.



too skewed outlier

Based on this histogram, we would conclude that
A) the 95% confidence interval computed from these data is very reliable.
B) the 95% confidence interval computed from these data is not very reliable.
C) the 95% confidence interval computed from these data is actually a 99% confidence interval.
D) the 95% confidence interval computed from these data is actually a 90% confidence interval.
E) only one student's score should fall outside the 95% confidence interval.

4. A 90% confidence interval for the mean μ of a population is computed from a random sample and is found to be 9 ± 3. Which of the following *could* be the 95% confidence interval based on the same data?
A) 9 ± 1.96.
B) 9 ± 2.
C) 9 ± 3.
D) 9 ± 4.
E) Without knowing the sample size, any of the above answers could be the 95% confidence interval.

higher Error for 95%

5. An agricultural researcher plants 25 plots with a new variety of corn. The average yield for these plots is $\bar{X} = 150$ bushels per acre. Assume that the yield per acre for the new variety of corn follows a normal distribution with unknown mean μ and standard deviation $\sigma = 10$ bushels. A 90% confidence interval for μ is
A) 150 ± 2.00. B) 150 ± 3.29. C) 150 ± 3.92. D) 150 ± 16.45.
E) 150 ± 32.90.

$1.64\left(\frac{10}{\sqrt{25}}\right)$

6. An agricultural researcher plants 25 plots with a new variety of corn. A 90% confidence interval for the average yield for these plots is found to be 162.72 ± 4.47 bushels per acre. Which of the following would produce a confidence interval with a smaller margin of error than this 90% confidence interval?
   A) Choosing a sample with a larger standard deviation.
   B) Planting 100 plots, rather than 25.
   C) Choosing a sample with a smaller standard deviation.
   D) Planting only 5 plots, rather than 25.
   E) None of the above.

Smaller ME

90 → sample

Use the following to answer questions 7 and 8:

You measure the heights of a random sample of 400 high school sophomore males in a Midwestern state. The sample mean is $\bar{X} = 66.2$ inches. Suppose that the heights of all high school sophomore males follow a normal distribution with unknown mean μ and standard deviation $\sigma = 4.1$ inches.

$66.2 \pm 1.96\left(\frac{4.1}{\sqrt{400}}\right)$

$.4018$

7. A 95% confidence interval for μ (expressed in interval notation) is
   A) (58.16, 74.24). B) (59.46, 72.94). C) (65.8, 66.6). D) (65.86, 66.54).
   E) (66.18, 66.22).

8. I compute a 95% confidence interval for μ. Suppose I had measured the heights of a random sample of 100 sophomore males, rather than 400. Which of the following statements is true?
   A) The margin of error for our 95% confidence interval would increase.
   B) The margin of error for our 95% confidence interval would decrease.
   C) The margin of error for our 95% confidence interval would stay the same, since the level of confidence has not changed.
   D) σ would increase.
   E) σ would decrease.

ME ↑

9. Suppose that the population of the scores of all high school seniors who took the Math SAT (SAT-M) test this year follows a normal distribution with mean μ and standard deviation $\sigma = 100$. You read a report that says, "On the basis of a simple random sample of 100 high school seniors that took the SAT-M test this year, a confidence interval for μ is 512.00 ± 25.76." The confidence level for this interval is
   A) 90%. B) 95%. C) 96%. D) 99%. E) over 99.9%.

$25.76 = z_c\left(\frac{100}{\sqrt{100}}\right)$

$25.76 = z_c(10)$

$x = 2.576 \rightarrow z_c$

10. An agricultural researcher plants 25 plots with a new variety of corn. The average yield for these plots is $\bar{X} = 150$ bushels per acre. Assume that the yield per acre for the new variety of corn follows a normal distribution with unknown mean μ and that a 90% confidence interval for μ is found to be 150 ± 3.29. What can we deduce about the standard deviation σ of the yield per acre for the new variety of corn?
    A) σ is about 10.
    ~~B)~~ σ will be larger than if we used 100 plots.
    ~~C)~~ σ is 3.29.
    D) If we repeated the experiment many, many additional times and from each computed a 90% confidence interval, σ would be within 3.29 of the mean in approximately 90% of the intervals.
    E) σ will vary depending on the sample size.

$3.29 = 1.64\left(\frac{\sigma}{\sqrt{25}}\right)$ $\sigma = 10.03$

11. To assess the accuracy of a laboratory scale, a standard weight that is known to weigh 1 gram is repeatedly weighed a total of n times, and the mean $\bar{X}$ of the n weighings is computed. Suppose the scale readings are normally distributed with unknown mean μ and standard deviation $\sigma = 0.01$ grams. How large should n be so that a 95% confidence interval for μ has a margin of error of $\pm$ 0.0001?
    A) 100. B) 196. C) 385. D) 10,000. E) 38,416.

$n = \left(\frac{(1.96)(.01)}{.0001}\right)^2 = 38416$

12. The distribution of a critical dimension of crankshafts produced by a manufacturing plant for a certain type of automobile engine is normal with mean μ and standard deviation $\sigma = 0.02$ millimeters. Suppose I select a simple random sample of four of the crankshafts produced by the plant and measure this critical dimension. The results of these four measurements, in millimeters, are
    200.01 199.98 200.00 200.01
    Based on these data, a 90% confidence interval for μ is
    A) 200.00 ± 0.00082.
    B) 200.00 ± 0.00115.
    C) 200.00 ± 0.001645.
    D) 200.00 ± 0.00196.
    E) 200.00 ± 0.01645.

.05 .5 90% .05 (use z B/c normal

$1.645\left(\frac{.02}{\sqrt{4}}\right)$

13. The heights of young American women are normally distributed with mean μ and standard deviation $\sigma = 2.4$ inches. I select a simple random sample of four young American women and measure their heights in inches. The four heights are
    63 69 62 66
    Based on these data, a 99% confidence interval for μ is
    A) 65.00 ± 1.27. B) 65.00 ± 1.55. C) 65.00 ± 2.35. D) 65.00 ± 3.09.
    E) 65.00 ± 4.07.

.005 .5 99 .5

$2.576\left(\frac{2.4}{\sqrt{4}}\right)$

14. The heights of young American women are normally distributed with mean μ and standard deviation $\sigma = 2.4$ inches. If I want the margin of error for a 99% confidence interval for μ to be $\pm$ 1 inch, I should select a simple random sample of size
    A) 2. B) 7. C) 16. D) 38. E) 39.

$n = \left(\frac{(2.576)(2.4)}{1}\right)^2 = 38.2$

Round up $= 39$

15. The scores of a certain population on the Wechsler Intelligence Scale for Children (WISC) are thought to be normally distributed with mean μ and standard deviation $\sigma = 10$. A simple random sample of 25 children from this population is taken, and each child is given the WISC. The mean of the 25 scores is $\bar{X} = 104.32$. Based on these data, a 95% confidence interval for μ is

A) 104.32 ± 0.78.
B) 104.32 ± 1.04.
C) 104.32 ± 3.29.
D) 104.32 ± 3.92.
E) 104.32 ± 19.60.

$1.96\left(\frac{10}{\sqrt{25}}\right)$

16. Suppose we want to compute a 90% confidence interval for the average amount μ spent on books by freshmen in their first year at a major university. The interval is to have a margin of error of \$2, and the amount spent has a normal distribution with standard deviation $\sigma = \$30$. The number of observations required is closest to

A) 25. B) 30. C) 608. D) 609. E) 865.

$n = \left(\frac{(1.645)(30)}{2}\right)^2$

$n = 608.9$

17. Other things being equal, the margin of error of a confidence interval increases as

A) the sample size increases.
B) the sample mean increases.
C) the population standard deviation increases.
D) the confidence level decreases.
E) none of the above.

$t^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ $t^*\left(\frac{s}{\sqrt{n}}\right)$

18. Researchers are studying the yield of a crop in two locations. The researchers are going to compute independent 90% confidence intervals for the mean yield μ at each location. The probability that *at least one* of the intervals will cover the true mean yield at its location is

A) 0.19. B) 0.81. C) 0.90. D) 0.95. E) 0.99.

SKIP

19. A procedure for approximating sampling distributions (which can then be used to construct confidence intervals) when theory cannot tell us their shape is

A) least squares. B) the bootstrap. C) residual analysis. D) normalization.
E) standardization.

20. A small New England college has a total of 400 students. The Math SAT (SAT-M) score is required for admission. The mean SAT-M score of all 400 students is 640, and the standard deviation of SAT-M scores for all 400 students is 60. The formula for a 95% confidence interval yields the interval 640 ± 5.88. We may conclude that

A) 95% of all student Math SAT scores will be between 634.12 and 645.88.
B) if we repeated this procedure many, many times, only 5% of the 95% confidence intervals would fail to include the mean SAT-M score of the population of all students at the college.
C) 95% of the time, the population mean will be between 634.12 and 645.88.
D) the interval is incorrect; it is much too small.
E) none of the above is true.

not exact