Chapter 2

2.1 Eleanor's standardized score, $z = \frac{680 - 500}{100} = 1.8$, is higher than Gerald's standardized score, $z = \frac{27 - 18}{6} = 1.5$.

2.2 The standardized batting averages (*z*-scores) for these three outstanding hitters are:

Player	*z*-score
Cobb	$z = \frac{.420 - .266}{.0371} = 4.15$
Williams	$z = \frac{.406 - .267}{.0326} = 4.26$
Brett	$z = \frac{.390 - .261}{.0317} = 4.07$

All three hitters were at least 4 standard deviations above their peers, but Williams' *z*-score is the highest.

2.3 (a) Judy's bone density score is about one and a half standard deviations below the average score for all women her age. The fact that your standardized score is negative indicates that your bone density is below the average for your peer group. The magnitude of the standardized score tells us how many standard deviations you are below the average (about 1.5). (b) If we let σ denote the standard deviation of the bone density in Judy's reference population, then we can solve for σ in the equation $-1.45 = \frac{948 - 956}{\sigma}$. Thus, $\sigma \doteq 5.52$.

2.4 (a) Mary's *z*-score (0.5) indicates that her bone density score is about half a standard deviation above the average score for all women her age. Even though the two bone density scores are exactly the same, Mary is 10 years older so her *z*-score is higher than Judy's (−1.45). Judy's bones are healthier when comparisons are made to other women in their age groups. (b) If we let σ denote the standard deviation of the bone density in Mary's reference population, then we can solve for σ in the equation $0.5 = \frac{948 - 944}{\sigma}$. Thus, $\sigma \doteq 8$. There is more variability in the bone densities for older women, which is not surprising.

2.5 (a) A histogram is shown below. The distribution of unemployment rates is symmetric with a center around 5%, rates varying from 2.7% to 7.1%, and no gaps or outliers.

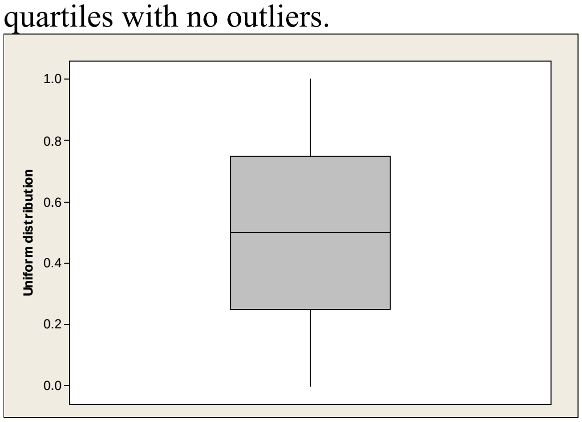
$z = \dfrac{64 - 46.9}{10.9} \doteq 1.57$ among the national group and $z = \dfrac{64 - 58.2}{9.4} \doteq 0.62$ among the 50 boys at his school. (c) The boys at Scott's school did very well on the PSAT. Scott's score was relatively better when compared to the national group than to his peers at school. Only 5.2% of the test takers nationally scored 65 or higher, yet about 23.47% scored 65 or higher at Scott's school. (d) Nationally, at least 89% of the scores are between 20 and 79.6, so at most 11% score a perfect 80. At Scott's school, at least 89% of the scores are between 30 and 80, so at most 11% score 29 or less.

2.8 Larry's wife should gently break the news that being in the 90th percentile is not good news in this situation. About 90% of men similar to Larry have identical or lower blood pressures. The doctor was suggesting that Larry take action to lower his blood pressure.

2.9 Sketches will vary. Use them to confirm that the students understand the meaning of (a) symmetric and bimodal and (b) skewed to the left.

2.10 (a) The area under the curve is a rectangle with height 1 and width 1. Thus, the total area under the curve is 1×1 = 1. (b) The area under the uniform distribution between 0.8 and 1 is 0.2×1 = 0.2, so 20% of the observations lie above 0.8. (c) The area under the uniform distribution between 0 and 0.6 is 0.6×1 = 0.6, so 60% of the observations lie below 0.6. (d) The area under the uniform distribution between 0.25 and 0.75 is 0.5×1 = 0.5, so 50% of the observations lie between 0.25 and 0.75. (e) The mean or "balance point" of the uniform distribution is 0.5.

2.11 A boxplot for the uniform distribution is shown below. It has equal distances between the quartiles with no outliers.



2.12 (a) Mean C, median B; (b) mean A, median A; (c) mean A, median B.

2.13 (a) The curve satisfies the two conditions of a density curve: curve is on or above horizontal axis, and the total area under the curve = area of triangle + area of 2 rectangles = $\frac{1}{2} \times 0.4 \times 1 + 0.4 \times 1 + 0.4 \times 1 = 0.2 + 0.4 + 0.4 = 1$. (b) The area under the curve between 0.6 and 0.8 is 0.2×1 = 0.2. (c) The area under the curve between 0 and 0.4 is

from $z = \frac{8000000 - 4243283.33}{5324827.26} \doteq 0.71$ in 2004 to 0.79 in 2005. Damon's salary percentile increased from the 87th (26 out of 30) in 2004 to the 93rd (26 out of 28) in 2005, while McCarty's decreased from the 20th (6 out of 30)in 2004 to the 14th (4 out of 28) in 2005.

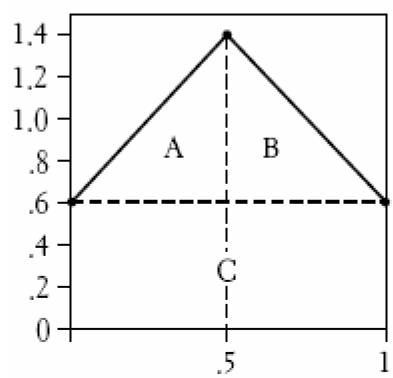
2.18 (a) The intervals, percents guaranteed by Chebyshev's inequality, observed counts, and observed percents are shown in the table below.

k	Interval	% guaranteed by Chebyshev	Number of values in interval	Percent of values in interval
1	73.93−86.07	At least 0%	18	72%
2	67.86−92.14	At least 75%	23	92%
3	61.79−98.21	At least 89%	25	100%
4	55.72−104.28	At least 93.75%	25	100%
5	49.65−110.35	At least 96%	25	100%

As usual, Chebyshev's inequality is very conservative; the observed percents for each interval are higher than the guaranteed percents. (b) Each student's z-score and percentile will stay the same because all of the scores are simply being shifted up by 4 points, $z = \frac{(x+4)-(\overline{x}+4)}{s} = \frac{x-\overline{x}}{s}$. (c) Each student's z-score and percentile will stay the same because all of the scores are being multiplied by the same positive constant, $z = \frac{1.06x - 1.06\overline{x}}{1.06s} = \frac{x-\overline{x}}{s}$. (d) This final plan is recommended because it allows the teacher to set the mean (84) and standard deviation (4) without changing the overall position of the students.

2.19 (a) Erik had a relatively good race compared the other athletes who completed the state meet, but had a poor race by his own standards. (b) Erica was only a bit slower than usual by her own standards, but she was relatively slow compared to the other swimmers at the state meet.

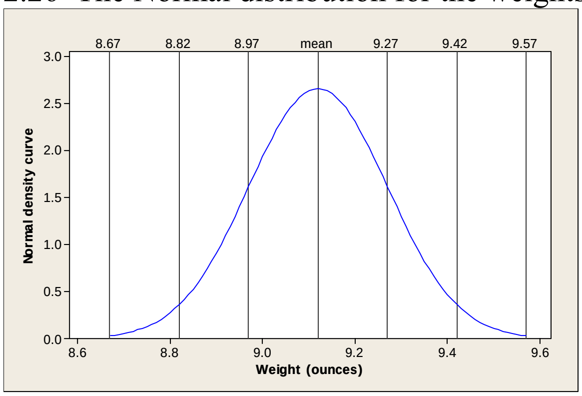
2.20 (a) The density curve is shown below.



The area under the density curve is equal to the area of A + B + C = $\frac{1}{2}\times 0.5\times 0.8 + \frac{1}{2}\times 0.5\times 0.8 + 1\times 0.6 = 1$. (b) The median is at $x = 0.5$, and the quartiles are at approximately $x = 0.3$ and $x = 0.7$. (c) The first line segment has an equation of $y = 0.6 + 1.6x$. Thus, the height of the density curve at 0.3 is $0.6 + 1.6\times 0.3 = 1.08$. The total area under the

one standard deviation below the mean. (d) The value 71.5 is one standard deviation above the mean. Thus, the area to the left of 71.5 is the 0.68 + 0.16 = 0.84. In other words, 71.5 is the 84th percentile of adult male American heights.

2.26 The Normal distribution for the weights of 9-ounce bags of potato chips is shown below.



The interval containing weights within 1 standard deviation of the mean goes from 8.97 to 9.27. The interval containing weights within 2 standard deviations of the mean goes from 8.82 to 9.42. The interval containing weights within 3 standard deviations of the mean goes from 8.67 to 9.57. (b) A bag weighing 8.97 ounces, 1 standard deviation below the mean, is at the 16th percentile. (c) We need the area under a Normal curve from 3 standard deviations below the mean to 1 standard above the mean. Using the 68−95−99.7 Rule, the area is equal to $0.68+\frac{1}{2}(0.95-0.68)+\frac{1}{2}(0.997-0.95)=0.8385$, so about 84% of 9-ounce bags of these potato chips weigh between 8.67 ounces and 9.27 ounces.

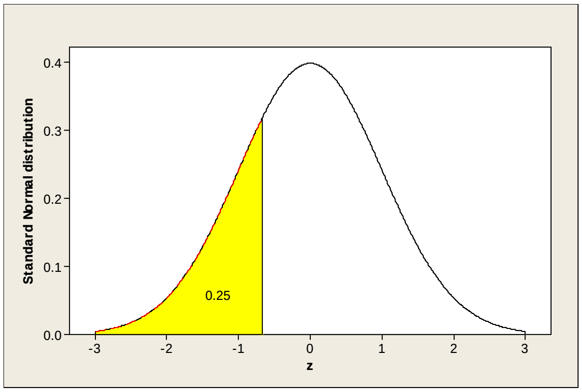
2.27 Answers will vary, but the observed percents should be close to 68%, 95%, and 99.7%.

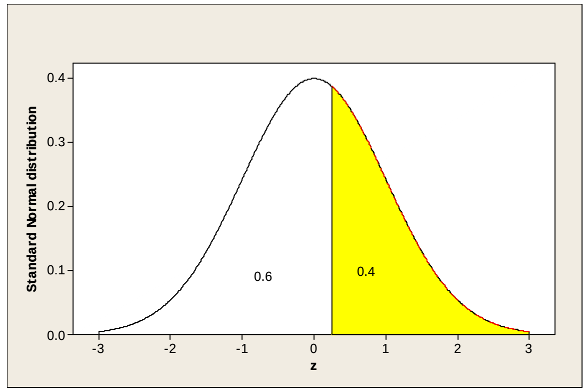
2.28 Answers will differ slightly from 68%, 95%, and 99.7% because of natural variation from trial to trial.

2.29 (a) 0.9978 (b) 1 − 0.9978 = 0.0022 (c) 1 – 0.0485 = 0.9515 (d) 0.9978 – 0.0485 = 0.9493

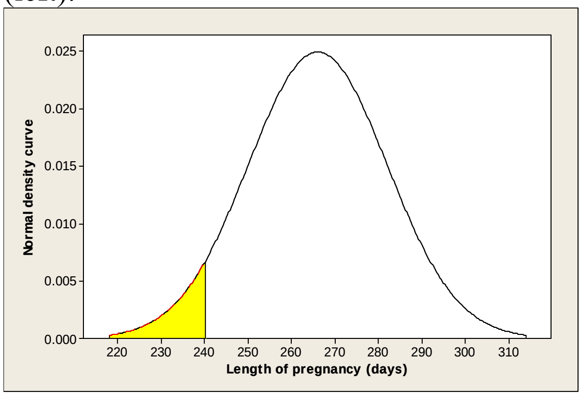
2.30 (a) 0.0069 (b) 1 − 0.9931 = 0.0069 (c) 0.9931 − 0.8133 = 0.1798 (d) 0.1020 − 0.0016 = 0.1004

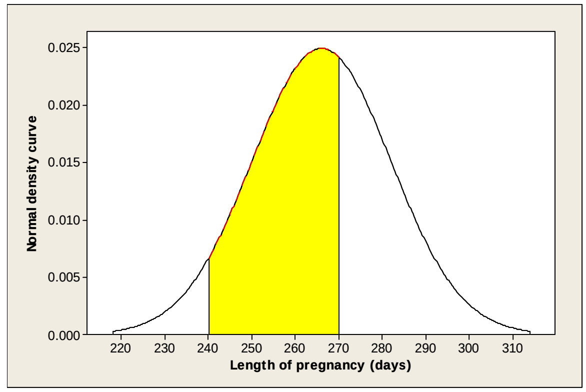
2.31 (a) We want to find the area under the N(0.37, 0.04) distribution to the right of 0.4. The graphs below show that this area is equivalent to the area under the N(0, 1) distribution to the right of $z=\frac{0.4-0.37}{0.04}=0.75$.



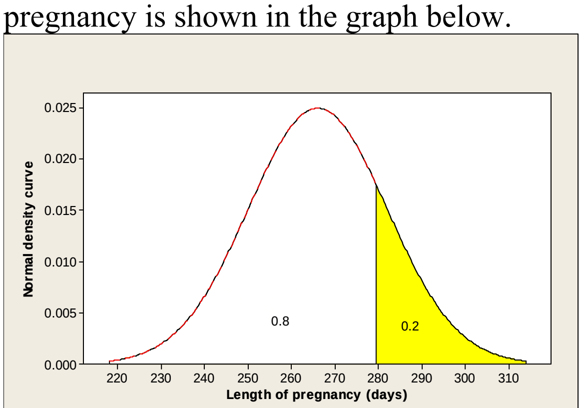


2.33 (a) The proportion of pregnancies lasting less than 240 days is shown in the graph below (left).



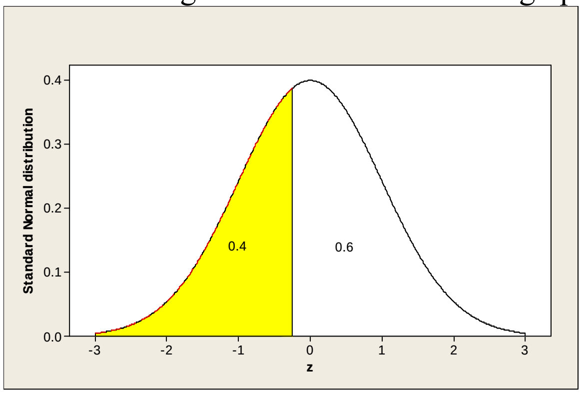


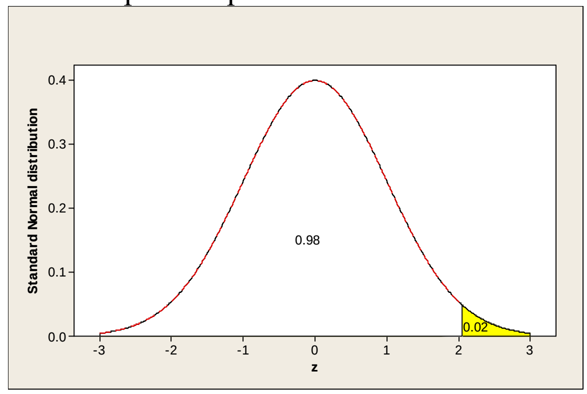
The shaded area is equivalent to the area under the N(0, 1) distribution to the left of $z = \frac{240-266}{16} \doteq -1.63$, which is 0.0516 or about 5.2%. (b) The proportion of pregnancies lasting between 240 and 270 days is shown in the graph above (right). The shaded area is equivalent to the area under the N(0, 1) distribution between $z = -1.63$ and $z = \frac{270-266}{16} = 0.25$, which is $0.5987 - 0.0516 = 0.5471$ or about 55%. (c) The 80th percentile for the length of human pregnancy is shown in the graph below.



Using Table A, the 80th percentile for the standard Normal distribution is 0.84. Therefore, the 80th percentile for the length of human pregnancy can be found by solving the equation

2.36 Use the given information and the graphs below to set up two equations in two unknowns.





The two equations are $-0.25=\frac{1-\mu}{\sigma}$ and $2.05=\frac{2-\mu}{\sigma}$. Multiplying both sides of the equations by σ and subtracting yields $-2.3\sigma=-1$ or $\sigma=\frac{1}{2.3}\doteq 0.4348$ minutes. Substituting this value back into the first equation we obtain $-0.25=\frac{1-\mu}{0.4348}$ or $\mu=1+0.25\times 0.4348\doteq 1.1087$ minutes.

2.37 Small and large percent returns do not fit a Normal distribution. At the low end, the percent returns are smaller than expected, and at the high end the percent returns are slightly larger than expected for a Normal distribution.

2.38 The shape of the quantile plot suggests that the data are right-skewed. This can be seen in the flat section in the lower left—these numbers were less spread out than they should be for Normal data—and the three apparent outliers that deviate from the line in the upper right; these were much larger than they would be for a Normal distribution.

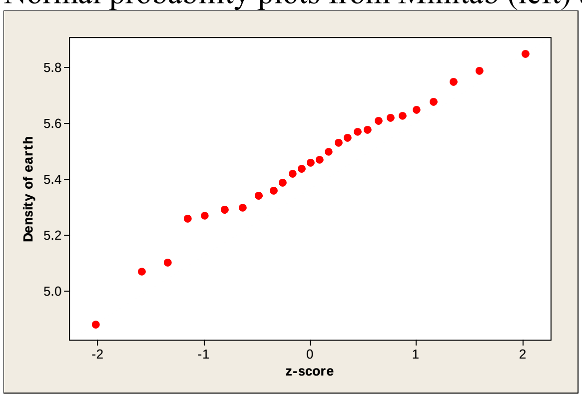
2.39 (a) *Who?* The individuals are great white sharks. *What?* The quantitative variable of interest is the length of the sharks, measured in feet. *Why?* Researchers are interested in the size of great white sharks. *When, where, how, and by whom?* These questions are impossible to answer based on the information provided. *Graphs:* A histogram and stemplot are provided below.

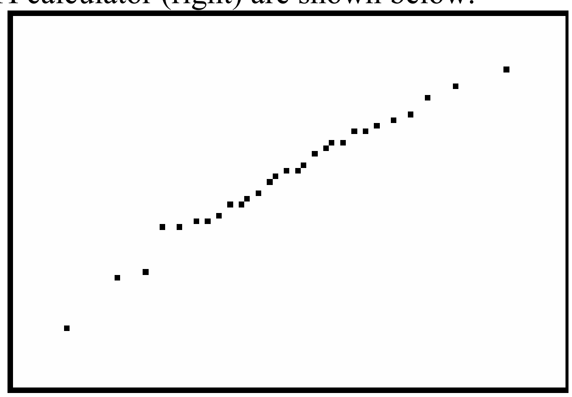
2.40 (a) A stemplot is shown below. The distribution is roughly symmetric.

```
Stem-and-leaf of density  N  = 29
Leaf Unit = 0.010

 1   48  8
 1   49
 2   50  7
 3   51  0
 7   52  6799
 12  53  04469
(4)  54  2467
 13  55  03578
 8   56  12358
 3   57  59
 1   58  5
```

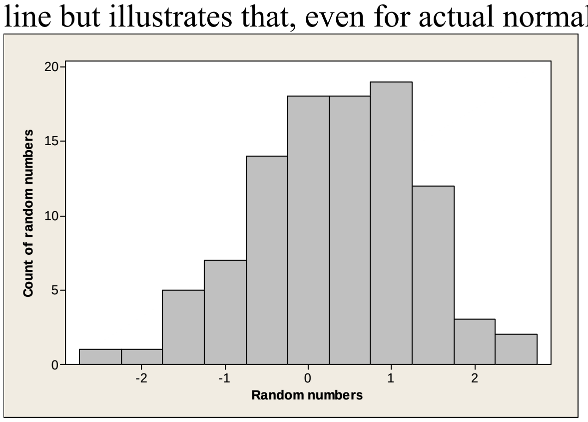
(b) The mean is $\bar{x} = 5.4479$ and the standard deviation is $s = 0.2209$. The densities follow the 68−95−99.7 rule closely—75.86% (22 out of 29) of the densities fall within one standard deviation of the mean, 96.55% (28 out of 29) of the densities fall within two standard deviations of the mean, and 100% of the densities fall within 3 standard deviations of the mean. (c) Normal probability plots from Minitab (left) and a TI calculator (right) are shown below.

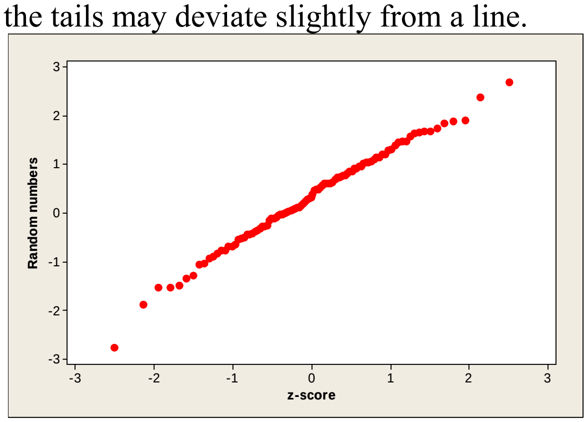




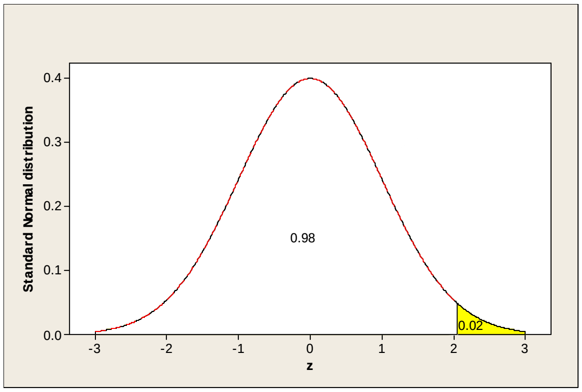
Yes, the Normal probability plot is roughly linear, indicating that the densities are approximately Normal.

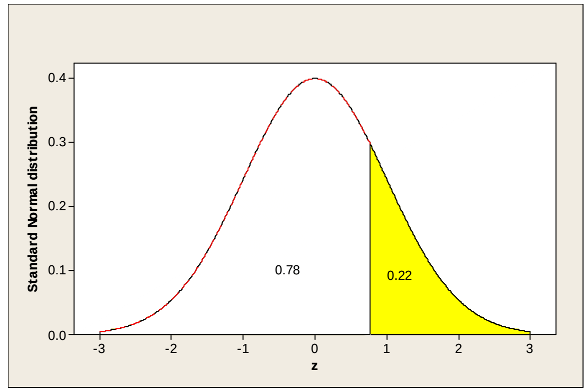
2.41 (a) A histogram from one sample is shown below. Histograms will vary slightly but should suggest a bell curve. (b) The Normal probability plot below shows something fairly close to a line but illustrates that, even for actual normal data, the tails may deviate slightly from a line.



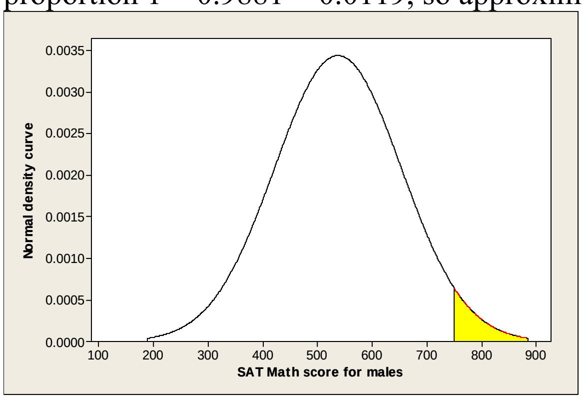


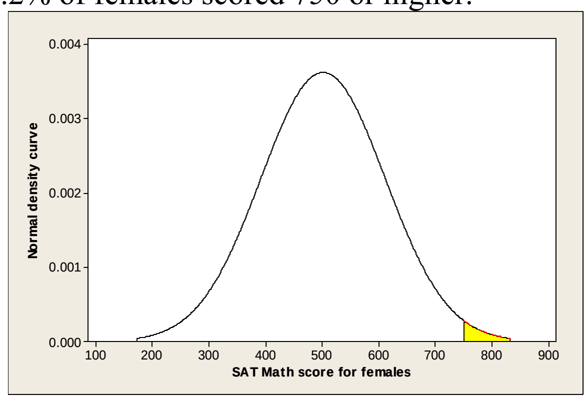
2.42 (a) A histogram from one sample is shown below. Histograms will vary slightly but should suggest the density curve of Figure 2.8 (but with more variation than students might expect).



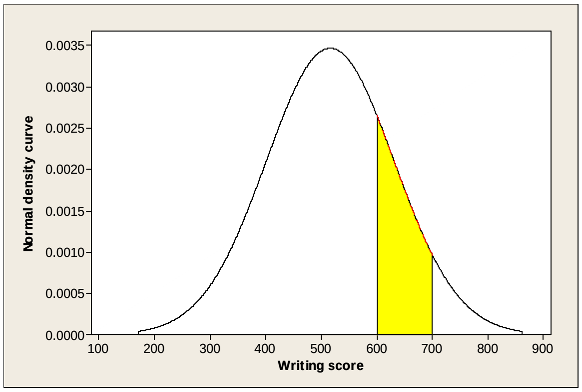


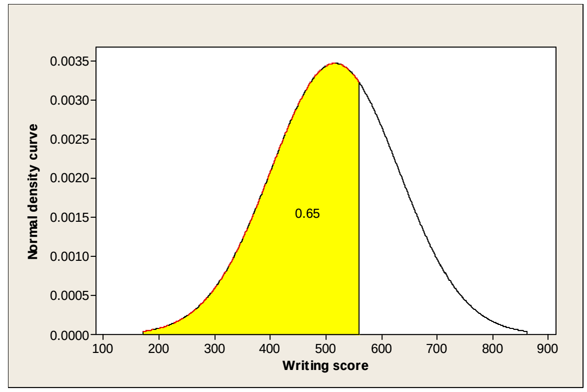
2.45 **(a)** To find the shaded area below for men, standardize the score of 750 to obtain the z-score of $z = \frac{750-537}{116} \doteq 1.84$. Table A gives the proportion 1 − 0.9671 = 0.0329, so approximately 3.3% of males scored 750 or higher. (b) For women, the shaded area below corresponds to getting a standardized score greater than $z = \frac{750-501}{110} \doteq 2.26$. Table A gives the proportion 1 − 0.9881 = 0.0119, so approximately 1.2% of females scored 750 or higher.





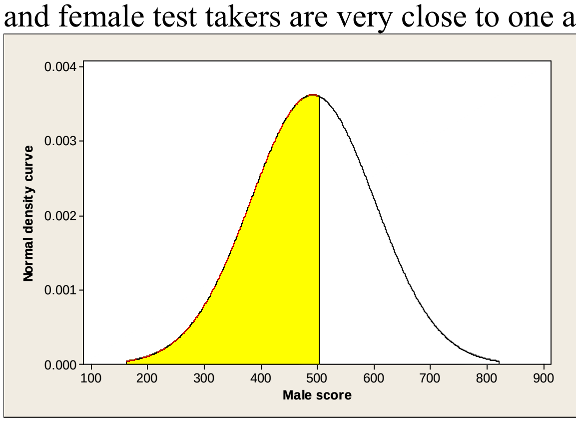
2.46 (a) According to the 68−95−99.7 rule, the middle 95% of all yearly returns are between 12 − 2×16.5 = −21% and 12 + 2×16.5 = 45%. (b) To find the shaded area below zero, indicated on the figure below, standardize 0 to obtain the z-score of $z = \frac{0-12}{16.5} \doteq -0.73$. Table A gives the proportion 0.2327 (software gives 0.233529). (c) To find the shaded area above 25%, indicated on the figure below, standardize 25 to obtain the z-score of $z = \frac{25-12}{16.5} \doteq 0.79$. Table A gives the proportion 1 − 0.7852 = 0.2148 (software gives 0.215384).

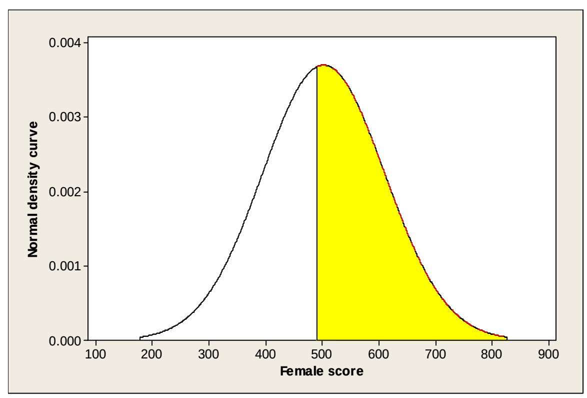




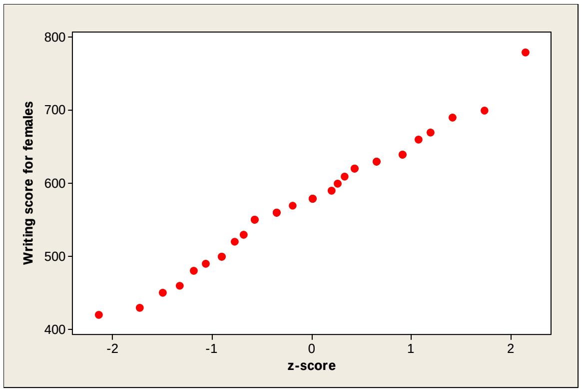
(b) The 65th percentile is shown above (right). Using Table A, the 65th percentile of a standard Normal distribution is closest to 0.39, so the 65th percentile for Writing score is 516 + 0.39×115 = 560.85.

2. (a) The proportion of male test takers who earned scores below 502 is shown below (left). Standardizing the score yields a *z*-score of $z = \dfrac{502 - 491}{110} = 0.10$. Table A gives the proportion 0.5398 or about 54%. (b) The proportion of female test takers who earned scores above 491 is shown below (right). Standardizing the score yields a *z*-score of $z = \dfrac{491 - 502}{108} \doteq -0.10$. Table A gives the proportion 1 − 0.4602 = 0.5398 or about 54%. (Minitab gives 0.5406.) The probabilities in (a) and (b) are almost exactly the same because the standard deviations for male and female test takers are very close to one another.

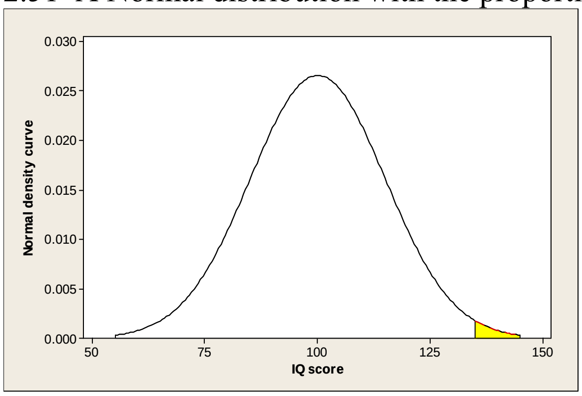




(c) The 85th percentile for the female test takers is shown below (left). Using Table A, the 85th percentile of the standard Normal distribution is closest to 1.04, so the 85th percentile for the female test takers is $502 + 1.04 \times 108 \doteq 614$. The proportion of male test takers who score above 614 is shown below (right). Standardizing the score yields a z-score of $z = \dfrac{614 - 491}{110} \doteq 1.12$. Table A gives the proportion 1 − 0.8686 = 0.1314 or about 13%.

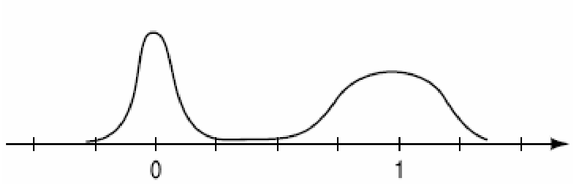


2.51 A Normal distribution with the proportion of "gifted" students is shown below.



A WISC score of 135 corresponds to a standardized score of $z = \frac{135-100}{15} \doteq 2.33$. Using Table A, the proportion of "gifted" students is 1 − 0.9901 = 0.0099 or .99%. Therefore, 0.0099×1300=12.87 or about 13 students in this school district are classified as gifted.

2.52 Sketches will vary, but should be some variation on the one shown below: The peak at 0 should be "tall and skinny," while near 1, the curve should be "short and fat."



2.53 The percent of actual scores at or below 27 is $\frac{1052490}{1171460} \times 100 \doteq 89.84\%$. A score of 27 corresponds to a standard score of $z = \frac{27-20.9}{4.8} \doteq 1.27$. Table A indicates that 89.8% of scores in a Normal distribution would fall below this level. Based on these calculations, the Normal distribution does appear to describe the ACT scores well.

2.54 (a) Joey's scoring "in the 97th percentile" on the reading test means that Joey scored as well as or better than 97% of all students who took the reading test and scored worse than about 3%. His scoring in the 72nd percentile on the math portion of the test means that he scored as

2.57 (a) The mean $\bar{x} = \$17,776$ is greater than the median M = \$15,532. Meanwhile, $M - Q_1 = \$5,632$ and $Q_3 - M = \$6,968$, so Q_3 is further from the median than Q_1. Both of these comparisons result in what we would expect for right-skewed distributions. (b) From Table A, we estimate that the third quartiles of a Normal distribution would be 0.675 standard deviations above the mean, which would be \$17,776 + 0.675 × \$12,034 ≐ \$25,899. (Software gives 0.6745, which yields \$25,893.) As the exercise suggests, this quartile is larger than the actual value of Q_3.

2.58 (a) About 0.6% of healthy young adults have osteoporosis (the area below a standard *z*-score of −2.5 is 0.0062). (b) About 31% of this population of older women has osteoporosis: The BMD level that is 2.5 standard deviations below the young adult mean would standardize to −0.5 for these older women, and the area to the left of this standard *z*-score is 0.3085.

2.59 (a) Except for one unusually high value, these numbers are reasonably Normal because the other points fall close to a line. (b) The graph is almost a perfectly straight line, indicating that the data are Normal. (c) The flat portion at the bottom and the bow upward indicate that the distribution of the data is right-skewed data set with several outliers. (d) The graph shows 3 clusters or mounds (one at each end and another in the middle) with a gap in the data towards the lower values. The flat sections in the lower left and upper right illustrate that the data have peaks at the extremes.

2.60 If the distribution is Normal, it must be symmetric about its mean—and in particular, the 10th and 90th percentiles must be equal distances below and above the mean—so the mean is 250 points. If 225 points below (above) the mean is the 10th (90th) percentile, this is 1.28 standard deviations below (above) the mean, so the distribution's standard deviation is $\frac{225}{1.28} \doteq 175.8$ points.

2.61 Use window of $X[55,145]_{15}$ and $Y[-0.008, 0.028]_{.01}$. (a) The calculator command shadeNorm(135,1E99,100,15) produces an area of 0.009815. About .99% of the students earn WISC scores above 135. (b) The calculator command shadeNorm(-1E99,75,100,15) produces an area of 0.04779. About 4.8% of the students earn WISC scores below 75. (c) shadeNorm(70,130,100,15) = 0.9545. Also, 1 – 2(shadeNorm(-1E99,70,100,15)) = 0.9545.

2.62 The calculator command normalcdf (−1E99, 27, 20.9, 4.8) produces an area of 0.89810596 or 89.81%, which agrees with the value obtained in Exercise 2.53.

2.63 The calculator commands invNorm(.05,22.8,1.1) = 20.99 and invNorm(.95,22.8,1.1) = 24.61 agree with the values obtained in Exercise 2.55.