

Name _____

TEACHER

Part 1: Multiple Choice. Circle the letter corresponding to the best answer.

- Suppose X is a random variable with mean μ . Suppose we observe X many times and keep track of the average of the observed values. The law of large numbers says that
 - the value of μ will get larger and larger as we observe X .
 - as we observe X more and more, this average and the value of μ will get larger and larger.
 - this average will get closer and closer to μ as we observe X more and more often.
 - as we observe X more and more, this average will get to be a larger and larger multiple of μ .
 - None of the above.
- In a population of students, the number of calculators owned is a random variable X with $P(X = 0) = 0.2$, $P(X = 1) = 0.6$, and $P(X = 2) = 0.2$. The mean of this probability distribution is

$$0(.2) + 1(.6) + 2(.2) = 0 + .6 + .4 = 1$$
 - 0.
 - 2.
 - 1.
 - 0.5.
 - The answer cannot be computed from the information given.
- Refer to the previous problem. The variance of this probability distribution is

$$(1-0)^2(.2) + (1-1)^2(.6) + (2-1)^2(.2) = .2 + 0 + .2 = .4$$
 - 1.
 - 0.63.
 - 0.5.
 - 0.4.
 - The answer cannot be computed from the information given.
- The weight of reports produced in a certain department has a Normal distribution with mean 60 g and standard deviation 12 g. What is the probability that the next report will weigh less than 45 g?

$$\text{normalcdf}(-\infty, 45, 60, 12) = .1056$$
 - 0.1042
 - 0.1056
 - 0.3944
 - 0.0418
 - The answer cannot be computed from the information given.

5. A randomly chosen subject arrives for a study of exercise and fitness. Consider these statements.

- I. After 10 minutes on an exercise bicycle, you ask the subject to rate his or her effort on the Rate of Perceived Exertion (RPE) scale. RPE ranges in whole-number steps from 6 (no exertion at all) to 20 (maximum exertion).
- II. You measure VO_2 , the maximum volume of oxygen consumed per minute during exercise. VO_2 is generally between 2.5 liters per minute and 6 liters per minute.
- III. You measure the maximum heart rate (beats per minute).

The statements that describe a discrete random variable are

- (a) None of the statements describes a discrete random variable.
- (b) I.
- (c) II.
- (d) I, III.
- (e) I, II, III.

$\frac{40}{52}$

6. A dealer in the Sands Casino in Las Vegas selects 40 cards from a standard deck of 52 cards. Let Y be the number of red cards (hearts or diamonds) in the 40 cards selected. Which of the following best describes this setting?

- (a) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$.
- (b) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$, provided the deck is shuffled well.
- (c) Y has a binomial distribution with $n = 40$ observations and probability of success $p = 0.5$, provided after selecting a card it is replaced in the deck and the deck is shuffled well before the next card is selected.
- (d) Y has a normal distribution with mean $p = 0.5$.
- (e) Y has a geometric distribution with $n = 40$ observations and probability of success $p = 0.5$.

red or
black
.5

7. A cell phone manufacturer claims that 92% of the cell phones of a certain model are free of defects. Assuming that this claim is accurate, how many cell phones would you expect to have to test until you find a defective phone?

- (a) 2, because it has to be a whole number
- (b) 8
- (c) 12.5
- (d) 92
- (e) 93

$$\frac{1}{0.08} =$$

8. Twenty percent of all trucks undergoing a certain inspection will fail the inspection. Assume that trucks are independently undergoing this inspection, one at a time. The expected number of trucks inspected before a truck fails inspection is

- (a) 2.
(b) 4.
(c) 5.
(d) 20.
(e) The answer cannot be computed from the information given.

9. Two percent of the circuit boards manufactured by a particular company are defective. If circuit boards are randomly selected for testing, the probability that the number of circuit boards inspected until a defective board is found is greater than 10 is

- (a) 1.024×10^7 .
(b) 5.12×10^7 .
(c) 0.1829.
(d) 0.8171.
(e) The answer cannot be computed from the information given.

$$P(x > 10) = 1 - \text{geom cdf}(.02, 10) = .8171$$

10. A random sample of 15 people is taken from a population in which 40% favor a particular political stand. What is the probability that exactly 6 individuals in the sample favor this political stand?

- (a) 0.6098
(b) 0.5000
(c) 0.4000
(d) 0.2066
(e) 0.0041

$$P(x=6) = \text{binom pdf}(15, .4, 6) = .2066$$

11. Experience has shown that a certain lie detector will show a positive reading (indicates a lie) 10% of the time when a person is telling the truth and 95% of the time when a person is lying. Suppose that a random sample of 5 suspects is subjected to a lie detector test regarding a recent one-person crime. Then the probability of observing no positive reading if all suspects plead innocent and are telling the truth is

- (a) 0.409.
(b) 0.735.
(c) 0.00001.
(d) 0.591.
(e) 0.99999.

$$\text{binom pdf}(5, .9, 5)$$

Part 2: Free Response Answer completely, but be concise. Write sequentially and show all steps.

10-1 5-2 3-5 1-10 1-100

12. A box contains ten \$1 bills, five \$2 bills, three \$5 bills, one \$10 bill, and one \$100 bill. A person is charged \$20 to select one bill.

20

(a) Identify the random variable. $X = 1, 2, 5, 10, 100$ dollar bills

$$X = -19$$

(b) Construct a probability distribution for these data.

X	1	2	5	10	100	Att
P(X)	.5	.25	.15	.05	.05	

(c) Find the expected value.

$$\mu = 1(.5) + 2(.25) + 5(.15) + 10(.05) + 100(.05)$$

$$= 7.25$$

(d) Is the game fair? Explain briefly. \rightarrow cost \$20 so gain potentially loss -12.75

No - you only have a chance to win/gain 5% of time
the probabilities vary. / you stand lose way too much money everytime you play

13. ACT scores for the 1,171,460 members of the 2004 high school graduating class who took the test closely followed the Normal distribution with mean 20.9 and standard deviation 4.8. Choose two students independently and at random from this group.

(a) What is the expected sum of their scores?

$$\mu_x + \mu_y = 20.9 + 20.9 = 41.8$$

(b) What is the expected difference of their scores?

$$\mu_x - \mu_y = 20.9 - 20.9 = 0$$

(c) What is the standard deviation of the difference in their scores?

$$\sigma = \sqrt{4.8^2 + 4.8^2} = \sqrt{46.08} = 6.8$$

can add var + then sq root.

(d) Find the probability that the sum of their scores is greater than 50.

$$P(X > 50)$$

$$\text{normalcdf}(50, \infty, 41.8, 6.8) = .1139$$



14. Patients receiving artificial knees often experience pain after surgery. The pain is measured on a subjective scale with possible values of 1 to 5. Assume that X is a random variable representing the pain score for a randomly selected patient. The following table gives part of the probability distribution for X .

X	1	2	3	4	5
$P(X)$	0.1	0.2	0.3	0.3	?

(a) Find $P(X = 5)$.

$$0.1$$

(b) Find the probability that the pain score is less than 3.

$$P(X < 3) = P(1) + P(2)$$

$$.1 + .2 = .3$$

(c) Find the mean μ for this distribution.

$$1(.1) + 2(.2) + 3(.3) + 4(.3) + 5(.1)$$

$$.1 + .4 + .9 + 1.2 + .5 = 3.1$$

(d) Find the standard deviation for this distribution.

$$\sigma^2 = (1-3.1)^2(.1) + (2-3.1)^2(.2) + (3-3.1)^2(.3) + (4-3.1)^2(.3) + (5-3.1)^2(.1)$$

$$\sigma = \sqrt{4.41 + .242 + .003 + .243 + .361} = \sqrt{5.259}$$

$$\sigma = 2.29$$

15. A headache remedy is said to be 80% effective in curing headaches caused by simple nervous tension. An investigator tests this remedy on 100 randomly selected patients suffering from nervous tension.

(a) Define the random variable being measured. $X = 0 - 100$
patients suffering

(b) What kind of distribution does X have? Justify your answer.

Binomial - BINS
✓✓✓✓

(c) Calculate the mean and standard deviation of X.

$$\mu = np = .80(100) = 80$$

$$\sigma = \sqrt{.8(100)(.2)} = \sqrt{16} = 4$$

(d) Determine the probability that exactly 80 subjects experience headache relief with this remedy.

$$P(X=80) = \text{binompdf}(100, .8, 80) = .0993$$

(e) What is the probability that between 75 and 90 (inclusive) of the patients will obtain relief? Justify your method of solution.

$$P(75 \leq X \leq 90) = P(X \leq 90) - P(X \leq 74)$$

$$\text{Binomcdf}(100, .8, 90) - \text{Binomcdf}(100, .8, 74)$$

$$.9977 - .0875 = .9102$$

I need 75 in there - knock out 74 + less

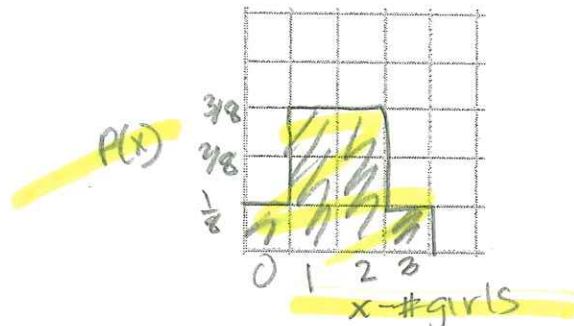


16. The Ferrells have three children: Jennifer, Jessica, and Jaclyn. If we assume that a couple is equally likely to have a girl or a boy, then how unusual is it for a family like the Ferrells to have three children who are all girls? Let X = number of girls (in a family of three children).

(a) Construct a pdf (probability distribution function) table for the variable X .

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(b) Construct a pdf histogram for X .



(e) What is the probability that a family like the Ferrells would have three children who are all girls?

$$P(X=3) = \frac{1}{8}$$

17. A survey conducted by the Harris polling organization discovered that 63% of all Americans are overweight. Suppose that a number of randomly selected Americans are weighed.

(a) Find the probability that 18 or more of the 30 students in a particular adult Sunday School class are overweight.

$$P(X \geq 18) = 1 - \text{binomcdf}(30, .63, 17)$$

$$= .7055$$

(b) How many Americans would you expect to weigh before you encounter the first overweight individual?

geom

$$\mu = \frac{1}{.63} = 1.59$$

(c) What is the probability that it takes more than 5 attempts before an overweight person is found?

$$P(X > 5) = 1 - \text{geomcdf}(.63, 5) = .0069$$

