AP PRACTICE CH 11-12

EXAMPLE

Healthy streams
Performing a significance test about μ

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PROBLEM: The level of dissolved oxygen (DO) in a stream or river is an important indicator of the water's ability to support aquatic life. A researcher measures the DO level at 15 randomly chosen locations along a stream. Here are the results in milligrams per liter (mg/l):

4.53	5.04	3.29	5.23	4.13	5.50	4.83	4.40
5.42	6.38	4.01	4.66	2.87	5.73	5.55	

An average dissolved oxygen level below 5 mg/l puts aquatic life at risk.

(a) Do the data provide convincing evidence at the $\alpha = 0.05$ significance level that aquatic life in this stream is at risk?

(b) Given your conclusion in part (a), which kind of mistake—a Type I error or a Type II error—could you have made? Explain what this mistake would mean in context.

$n = 15$

$H_0: \mu = 5$ The data has slight skewness (left) as seen

$H_a: \mu < 5$ By the Box plot. w/ $n = 15$, use t-test. (no pop st. Dev)

The parameter of interest is the mean DO level in a stream.

$\bar{x} = 4.77$

$S_x = .9396$

$t_\alpha = -1.76$

$t = \dfrac{4.77 - 5}{.9396/\sqrt{15}} = \dfrac{-.23}{.243} = -.9465$

There is insufficient evidence to reject the H_0 (null) $\mu = 5$ therefore we do not have evidence to support a claim that the DO level is less than 5 putting aquatic life at risk.

(b) Potially made a type II error. If we made a type II error ~~which~~ we would not support an insufficient DO level when in fact it may be unacceptable — this could result in Aquatic life being at risk

* STATE Ho + Ha

3. **How much juice?** One company's bottles of grapefruit juice are filled by a machine that is set to dispense an average of 180 milliliters (ml) of liquid. A quality-control inspector must check that the machine is working properly. The inspector takes a random sample of 40 bottles and measures the volume of liquid in each bottle.

$H_0: \mu = 180$

$H_a: \mu \neq 180$

4. **Attitudes** The Survey of Study Habits and Attitudes (SSHA) is a psychological test that measures students' attitudes toward school and study habits. Scores range from 0 to 200. Higher scores indicate better attitudes and study habits. The mean score for U.S. college students is about 115. A teacher suspects that older students have better attitudes toward school, on average. She gives the SSHA to an SRS of 45 of the over 1000 students at her college who are at least 30 years of age.

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5. **Cold cabin?** During the winter months, the temperatures at the Starneses' Colorado cabin can stay well below freezing (32°F or 0°C) for weeks at a time. To prevent the pipes from freezing, Mrs. Starnes sets the thermostat at 50°F. The manufacturer claims that the thermostat allows variation in home temperature of $\sigma = 3$°F. Mrs. Starnes suspects that the manufacturer is overstating the consistency of the thermostat.

$H_0: \sigma = 3$

$H_a: \sigma > 3$

6. **Ski jump** When ski jumpers take off, the distance they fly varies considerably depending on their speed, skill, and wind conditions. Event organizers must position the landing area to allow for differences in the distances that the athletes fly. For a particular competition, the organizers estimate that the variation in distance flown by the athletes will be $\sigma = 10$ meters. An experienced jumper thinks that the organizers are underestimating the variation.

In Exercises 7 and 8, explain what's wrong with the stated hypotheses. Then give correct hypotheses.

7. **Stating hypotheses**
   (a) A change is made that should improve student satisfaction with the parking situation at a local high school. Before the change, 37% of students approve of ~~the parking~~ that's provided. The null hypothesis $H_0: p > 0.37$ is tested against the alternative $H_a: \cancel{p = 0.37}$.

   $H_0 =$

   $H_a >$

   (b) A researcher suspects that the mean birth weights of babies whose mothers did not see a doctor before delivery is less than 3000 grams. The researcher states the hypotheses as

$$H_0: \overset{\mu}{\bar{x}} = 3000 \text{ grams}$$
$$H_a: \bar{x} < 3000 \text{ grams}$$

8. **Stating hypotheses**
   (a) A change is made that should improve student satisfaction with the parking situation at your school. Before the change, 37% of students approve of the parking that's provided. The null hypothesis $H_0: \hat{p} = 0.37$ is tested against the alternative $H_a: \hat{p} > 0.37$.

   (b) A researcher suspects that the mean birth weights of babies whose mothers did not see a doctor before delivery is less than 3000 grams. The researcher states the hypotheses as

$$H_0: \mu = 3000 \text{ grams}$$
$$H_a: \mu \leq 2999 \text{ grams}$$

9. **No homework?** Refer to Exercise 1. The math teachers inspect the homework assignments from a random sample of 50 students at the school. Only 68% of the students completed their math homework. A significance test yields a P-value of 0.1265.
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   (a) Explain what it would mean for the null hypothesis to be true in this setting.
   (b) Interpret the P-value.

10. **Attitudes** Refer to Exercise 4. In the study of older students' attitudes, the sample mean SSHA score was 125.7 and the sample standard deviation was 29.8. A significance test yields a P-value of 0.0101.
    (a) Explain what it would mean for the null hypothesis to be true in this setting.
    (b) Interpret the P-value.

11. **How much juice?** Refer to Exercise 3. The mean amount of liquid in the bottles is 179.6 ml and the standard deviation is 1.3 ml. A significance test yields a P-value of 0.0589. Interpret the P-value.

12. **Don't argue** Refer to Exercise 2. Yvonne finds that 96 of the 150 students (64%) say they rarely or never argue with friends. A significance test yields a P-value of 0.0291. Interpret the P-value.

13. **Interpreting a P-value** A student performs a test of $H_0: \mu = 100$ versus $H_a: \mu > 100$ and gets a P-value of 0.044. The student says, "There is a 0.044 probability of getting the sample result I did by chance alone." Explain why the student's explanation is wrong.

14. **Interpreting a P-value** A student performs a test of $H_0: p = 0.3$ versus $H_a: p < 0.3$ and gets a P-value of 0.22. The student says, "This means there is about a 22% chance that the null hypothesis is true." Explain why the student's explanation is wrong.

15. **No homework** Refer to Exercises 1 and 9. What conclusion would you make at the $\alpha = 0.05$ level?
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16. **Attitudes** Refer to Exercises 4 and 10. What conclusion would you make at the $\alpha = 0.05$ level?

packet

(#11) $P = .0589$ is relatively small → significantly small enough to reject $H_0 = 180$ and say its underfilling

(13) prob of getting a sample result at least as extreme as the one we got in the H_0 ($\mu = 100$) was true
so .044 or 4.4% chance of getting a # at least as Big as 100

(17) $\alpha = .10$ P value $.0589 < .10$ ⇒ Reject Null ($\mu = 180$)
(b) if $\alpha = .05$ my result would change to FTR H_0 B/K $.0589 \not< .05$.

(21) $H_0: \mu = 1$ $\bar{x} = .975$. The parameter of interest is the mean weight of all loaves of bread
$H_a: \mu < 1$
pvalue $= .0806$ $>$ $.0806 \not< .01$ ⇒ FTR H_0
$\alpha = .01$
There is insufficient evidence to Reject the H_0 → Bread mean $= 1$ and support the claim that they are too light at an $\alpha = .01$

(23) $H_0: \mu = \$85K$ A type I error would be to conclude a
$H_a: \mu > \$85K$ (test) false rejection of H_0 ($\mu = 85K$) and take the Alternative assuming income > 85K
this consequence would be opening an upsc restaurant in a neighborhood whose income cant support it ⇒ no business.

A type II error would be falsly failing to Rej H_0 and thus not opening the restaurant and thus missing out on an opportunity

$H_0: p = 12$
$H_a: p > 12$

Reject H_0 Falsly

(24) A Type I error would be concluding our alternative is correct when we should not have. This means we would air show but not get the viewers expected
A type II error would be falsly FTR H_0 and then not airing the show that would have been successful

(25) $H_0: p = .22$ $\alpha = .05$
$H_a: p < .22$
test if less than 22 %

Parameter is proportion of calls involving life threatening emergencies that took more than 8 min to arrive

Type I Error: Reject H_0 + should not have. If we reject and say its less than .22 (falsely) then proport = .22 and adjustments cant be made/ wont be made for better service + life threating response time is > 8 min for more people than thought.
Type II error: FTR H_0 + should have rejected it. If we dont Reject $p = .22$ but in fact its actually less than .22 then responders get possible repremand for not doing better.

(b) Type I worse in this case
(c) $\alpha = .05 \Rightarrow$ 5% chance making a type I error when is low. I think $.01 = \alpha$ would be better considering study to help save lives.

✓

(29) d (30) a (31) C Assuming H_0 true

(33) (a) P(Deg by wom) = .4168 $(.7)(.43) + (.24)(.41) + (.06)(.29)$
(24,611)(.4168) = 10,258
(b) not independent

(34) The mistake is in saying that 95% of other polls would would have results close to the result of the poll. other surveys should be close to the truth not necessarily close to the result of this survey

(35) Conditions

SRS ✓ Random 60

10% rule $60 \times 10 = 600 \rightarrow$ more than 600 at his "large urban school"

$np_0 \geq 10 \Rightarrow 60(.8) = 48 \geq 10$ ✓ $\}$ normal

$nq_0 \geq 10 \Rightarrow 60(.2) = 12 \geq 10$ ✓

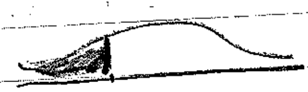
(38) conditions

random $\rightarrow$ no SRS - vol. response

$H_0: p = .80$ $H_a: p < .80$ $\hat{p} = \frac{41}{60} = .683$ $\alpha = .05$

(39) (a) $z = -2.26$

p value $= .0119$



$z = -1.645$

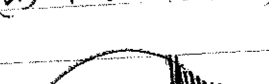
$.0119 < .05$

$z = -2.26 < z^* = 1.645$

$\rightarrow$ Reject H_0

sufficient evidence at $\alpha = .05$ to reject the null $p = .80$ and support claim $p < .80$

(41) (a) $P(z \geq 2.19) = 0.0143$



Right tailed test

b) $\alpha = .01$ $0.0143 \not< .01$ FTR H_0

If $\alpha = .05 \rightarrow$ Reject H_0

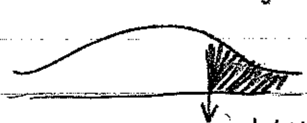
Decision changes B/c $.0143 < .05$

(c) $2.19 = \dfrac{\hat{p} - .5}{\sqrt{\dfrac{(.5)(.5)}{200}}} \Rightarrow \hat{p} = .5774$

(43) The parameter of interest is the true proportion of students (middle school) w/ bullying behavior.

Conditions
random ✓
10% rule ✓
$np_0 \geq 10$ ✓
$nq_0 \geq 10$ ✓

$H_0: p = .75$ $\alpha = .05$
$H_a: p > .75$ media claim



$z = 1.645$

$\hat{p} = \frac{445}{558} = .797$

$z = \frac{.797 - .75}{\sqrt{\frac{(.75)(.25)}{558}}} = 2.59 \rightarrow$ Reject Ho
pval = .004
.004 < .05 R-Ho

At $\alpha = .05$, there is suffic. evidence to reject the null $p = .75$ and support the claim that the true proportion of M.S. students w/Bullying behavior is greater than .75

(44) The parameter of interest is the ~~po~~ true proportion of seeds that will germinate

Conditions
SRS ✓
10% rule ✓
$np \geq 10 \Rightarrow .8(400)$ ✓
$nq \geq 10$ $.2(400)$ ✓

$H_0: p = .8$ $\alpha = .05$
$H_a: p > .8$ $\hat{p} = \frac{339}{400} = .8475$

$z = 1.645$

$z = \frac{.8475 - .8}{\sqrt{\frac{(.8)(.2)}{400}}} = 2.375 \rightarrow$ Reject Ho
pval = .0088
.0088 < .05

There is sufficient evidence @ $\alpha = .05$ to reject null that $p = .8$ and accept the alternative that the true proportion is greater than .8

(45) $H_0: p = .37$ $\hat{p} = \frac{83}{200} = .415$ if $\alpha = .05$ pvalue = .093 $.093 \nless .05 \rightarrow$ FTRH_0
H_a $p > .37$ claim

Type I error: Reject Ho & should not have. If we were to falsely reject Ho & accept claim, it would falsely show more than 37% approve the parking plan. Type II error: FTRHo & should have rejected it. If we dont reject Ho we would not support claim of $p_0 > .37$ when we should have. In this case, insuff. evidence to support claim based on $\alpha = .05$ (above

SRS ✓

✓ n 300 < 10% of 5000 Alz patients

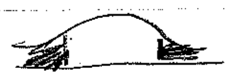
(46) $H_0: p = .10$ $\frac{25}{300} = \hat{p} = .083$ pval = .168

$H_a: p < .10$ $\alpha = .05$ $.168 \nless .05$ FTR H_0

Type I Error: Reject H_0 & should not have. In this case a Type I error would conclude that the $p = .10$ is not true for patience experiencing nausea & support comp claim that less than 10% Exp. nausea erroneously. They would dispense the med under false side effect assumption

Type II error would occur if we did not reject the .10 of patients exp. nausea & thus not support the manuf claim & ~~not~~ potentially not dispensing the drug treatment or see more people exp. nausea than thought

At a significance level of .05, we can not reject the H_0 so there is not enough evidence to support the claim.

(52) $H_0: p = .73$  $\hat{p} = .66$

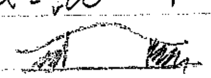
CONDIT $H_a: p \neq .73$ $\alpha = .05/2 = .025$ $p = .026$

w/o ruler ✓ $z^* = \pm 1.96$ $z = -2.23$ Reject

$n\hat{p}/n\hat{q} \geq 10$ At an $\alpha = .05$, there is sufficient evidence to

SRS ✓ reject the H_0 and support the claim that the proportion is diff. from the national value of .73

(58) $\alpha = .05$ $p = .011$ $z = 2.53$ $n = 439$ $x = 249$

 use $\hat{p} = .5$ $n\hat{p} = 439(.5) \geq 10$ ✓ } normal

$n\hat{q} = 439(.5) \geq 10$ ✓

parameter of interest is SRS & indep.

proportion of 13-17yo who feel they should wait til marriage

$p = .011 \Rightarrow$ is low and and $< .05 (\alpha)$ which is significant enough to reject H_0 and support claim that its different mean/prop of .56

(63) The distribution of X-Y has a normal distribution with $\mu_{X-Y} = \mu_X - \mu_Y = 5.3 - 5.26 = .04$ and Standard Deviation of $\sigma_{X-Y} = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{0.01^2 + .02^2} = .0224$. It is important that X-Y is a positive #

(b) $P(X-Y > 0) \Rightarrow$
normal cdf (0, ∞, .04, .0224) = .963 = 96.3% chance the mean diff > 0

(64) of the 10,065 → 3 groups

Employee gets	Emp gets	control
↓	↓	↓
3355	3355	3355
Grp 1	Gr 2	Gr 3

control group is there to see true impact pay vs w/ incentive pay / no incentive pay

~~Random~~ Random # generator
→ use # 1,2,3. go down list + use next # to assign the person to a group.

(66) SRS ✓
✓ 10% rule: 75(10) = 750 likely to dispense more than 750 bags in a day
sample ≥ 30 ⇒ normal.

(68) (a) The parameter of interest is the proportion of invoices that contain items that ~~#~~ can be found lower than what charging.

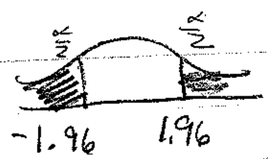
$H_0: p = .5$ $\quad \frac{20}{25}$ $\quad n\hat{p}$ 25(.8) ≥ 10 ✓
$H_a: p < .5$ $\quad n\hat{q}$ 25(.2) ≥ 10 ✗ > not normal strong skew right

SRS ✓ 25(10) = 250 ✓

conditions not met

* T-test.

(69) $H_0: \mu = 64$ $H_a: \mu \neq 64$ $\alpha = .05$



$t = \frac{62.8 - 64}{5.36/\sqrt{25}} = -1.12$

FTReject H_0

p-val = .137 × 2 = .2738

B/c given/calculated mean is less than (different from) 64, it may support Alternative. This was found to not be supported based on t-test

(72) $z = \frac{19.28 - 19.2}{.81/\sqrt{75}} = .889$

$\alpha = .10/2$

$p = .051 \times 2 = .101 \not< .1$

<u>conclusion</u>: There is insufficient evidence to supp. claim w/ $\alpha = .1$. Pvalue = .101 > .01 therefore we fail to reject the null that the true mean is 19.2 + support the claim that it is different from that

(74) $H_0: \mu_d = 0$
$H_a: \mu_d > 0$ claim
(Aft - bef)

The parameter of interest is the mean difference of additional hours people got w/ drug.

L1 → data T-test (#2)
$t = 3.67$ $p = .002$ $.002 < .01 \rightarrow$ Reject H_0.
$\bar{X} = 2.33$ $Sx = 2.00$

There is sufficient evidence @ $\alpha = .01$ to support the claim that the drug will offer ~~...~~ insomniacs more sleep. Possible Type I error (P(TypeI) = .01), which could mean that insomniacs will take the drug and not get the anticipated extra sleep.

(7) $H_0: \mu = 11.5$ μ is the true mean hardness
$H_a: \mu \neq 11.5$ of tablets using $\alpha = .05$
use t-test ($n=20$)
$t = .77$ 10% rule, normal → check graph for
$p = .4494$ skewness/outliers.
$.4494 \nless .05$ → FTR H_0 → can not reject μ hardness of 11.5
no evidence to support the alternative that it is ~~less than~~ different from 11.5.

Mult. Choice

(1) B (2) e (3) c (4) e (5) b (6) c

(7)